Quiz Wed Nov 1 Section 27.3

25.1 Antiderivatives Cont'd

$f'(x)$	$f(x)$
6	$6x$
x^4	$\frac{x^5}{5}$
$\frac{1}{x^3} = x^{-3}$	$-\frac{1}{2}x^{-2}$
$\sqrt[3]{x} = x^{1/3}$	$\frac{3}{4}x^{4/3}$
$8x^4$	$\frac{8x^5}{5}$

Ex: Find an antiderivative:

e) $f'(x) = \frac{1}{\sqrt{x}} + \frac{8}{3}\cdot\sqrt[4]{x} + \pi^2$

$= x^{-1/2} + \frac{8}{3}x^{1/4} + \pi^2 \leftarrow$ constant

$f(x) = 2x^{1/2} + \frac{8}{3}\left(\frac{4}{5}x^{\frac{5}{4}}\right) + \pi^2 x$

$= 2x^{1/2} + \frac{32}{15}x^{5/4} + \pi^2 x$

f) $f'(x) = 8(x^5+3)^7(5x^4)$

Guess and check.

$$g(x) = (x^5+3)^8$$

$$g'(x) = 8(x^5+3)^7(5x^4) \checkmark$$

$$f(x) = (x^5+3)^8$$

25.2 Indefinite Integrals

The integral $\int f(x)\,dx$ represents all possible antiderivatives of $f(x)$.

$$\boxed{\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \qquad (n \neq -1)}$$

any constant

$$\int x^3\,dx = \frac{x^4}{4} + C$$

$$\int 1\,dx = \int x^0\,dx = \frac{x^1}{1} + C = x + C$$

$$\int 7x^3\,dx = \frac{7x^4}{4} + C$$

$$\int 6\,dx = 6x + C$$

<u>Ex</u>: Evaluate

a) $\int (5x^3 + 6x)\,dx$

$$= \frac{5x^4}{4} + \frac{6x^2}{2} + C \quad \checkmark$$

$$= \frac{5x^4}{4} + 3x^2 + C \quad \checkmark$$

b) $\int \left(\frac{1}{r^4} + \frac{1}{r^3} + 8\right) dr$

$$= \int (r^{-4} + r^{-3} + 8)\,dr$$

$$= -\frac{1}{3}r^{-3} - \frac{1}{2}r^{-2} + 8r + C$$

c) $\int \left(\sqrt[3]{x} + \frac{1}{x^3}\right) dx$

$$= \int (x^{1/3} + x^{-3})\,dx$$

$$= \frac{3}{4}x^{4/3} - \frac{1}{2}x^{-2} + C$$

Substitution

Ex: Evaluate $\int x^4 (6x^5+13)^2 \, dx$

u^2

$\frac{du}{30}$

$u = 6x^5 + 13$

$\frac{du}{dx} = 30x^4$

$du = 30x^4 \, dx$

$\frac{du}{30} = x^4 \, dx$

$$= \int \frac{1}{30} u^2 \, du$$

$$= \frac{1}{30} \frac{u^3}{3} + C$$

$$= \frac{1}{90} (6x^5+13)^3 + C$$

Ex: Evaluate $\int \frac{11x}{\sqrt{x^2+4}} \, dx$

$u = x^2 + 4$

$\frac{du}{dx} = 2x$

$du = 2x \, dx$

$\frac{du}{2} = x \, dx$

$$= \int \frac{11}{2} \frac{1}{\sqrt{u}}\, du$$

$$= \frac{11}{2} \int u^{-1/2}\, du$$

$$= \frac{11}{2}\left(2u^{1/2} \right) + C$$

$$= 11\sqrt{x^2+4} + C$$

Ex: Evaluate $\displaystyle\int \frac{\overset{2(2z-3)}{\cancel{4z-6}}}{\sqrt{z^2-3z}}\, dz$

$$\boxed{\begin{aligned} u &= z^2-3z \\ \frac{du}{dz} &= 2z-3 \\ du &= (2z-3)\, dz \end{aligned}}$$

$$= \int \frac{2\, du}{\sqrt{u}}$$

$$= 2\int u^{-1/2}\, du$$

$$= 2\left(2u^{1/2} \right) + C$$

$$= 4\sqrt{z^2 - 3z} + C$$