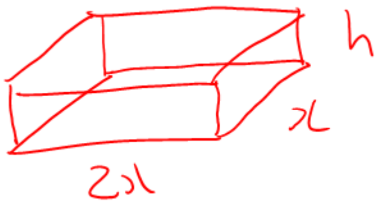
Test Review

Ex: A rectangular box has length = 2(width). The 3 dimensions sum to 36 cm. Find the maximum volume.



$$x + 2x + h = 36$$

Maximize $V = 2x(x)(h)$

$h = 36 - 3x \rightarrow V = 2x^2(36 - 3x)$

$$V = 72x^2 - 6x^3$$

$$V' = 144x - 18x^2$$

Set $V' = 0$: $0 = 144x - 18x^2$

$$0 = 18x(8 - x)$$

$x = 0$ MIN $\quad x = 8$ MAX

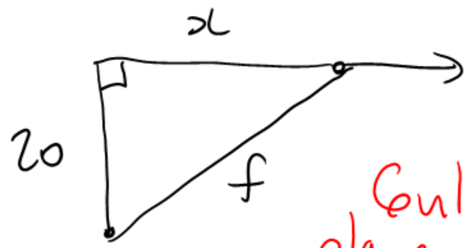
$$V\Big|_{x=8} = 2x^2(36 - 3x)\Big|_{x=8}$$

$$= 2(64)(12)$$

$= 1536 \quad cm^3$

Ex: Given $f = \sqrt{400 + x^2}$ and $\frac{dx}{dt} = 200 \frac{km}{h}$.

How fast is f changing when $x = 50$ km?



Could be a plane flying horizontally

$$f = (400 + x^2)^{1/2}$$

$$\frac{dx}{dt} = 200 \frac{km}{h}$$

$$\frac{df}{dt} = ?$$

$$x = 50$$

$$\frac{df}{dt} = \frac{df}{dx} \frac{dx}{dt} \qquad \text{Chain Rule}$$

$$= \frac{1}{2}(400 + x^2)^{-1/2}(2x)\frac{dx}{dt}$$

$$= \frac{1}{2}(400 + 50^2)^{-1/2}(100)(200)$$

$\approx 186 \ \frac{km}{h}$

Ex: Find $\frac{dy}{dx}$

a) $y = 11 \cos x^6$

$$\frac{dy}{dx} = 11(-\sin x^6)(6x^5)$$

$$= -66x^5 \sin x^6$$

b) $y = \sec^4(1+x^2)$

$$y = [\sec(1+x^2)]^4$$

$$\frac{dy}{dx} = 4[\sec(1+x^2)]^3 [\sec(1+x^2)\tan(1+x^2)(2x)]$$

$$= 8x \sec^4(1+x^2)\tan(1+x^2)$$

Ex: Position in m after t seconds:

$x = 3 - t^2 \qquad y = 16\sqrt{t} \leftarrow y = 16t^{1/2}$

Find the velocity at $t = 4$ s.

$v_x = -2t \qquad v_y = 8t^{-1/2}$

@ $t=4$: $v_x = -8 \qquad v_y = \frac{8}{\sqrt{4}} = 4$

speed $v = \sqrt{(-8)^2 + (4)^2}$

≈ 8.9 m/s

direction $\theta = \tan^{-1}\left(\frac{4}{-8}\right)$ (+ 180°?)

$V_x < 0$

$\approx -27° + 180°$

$\approx 153°$

<u>Ex</u>: Use differentials to estimate the change in $y = \frac{1}{x}$ as x goes from $x = 1$ to $x = 0.95$

$y = x^{-1}$

$$\frac{dy}{dx} = -x^{-2}$$

$$dy = -x^{-2}\,dx$$

$x = 1$

$dx = 0.95 - 1$

$= -0.05$

$$dy = -\frac{1}{1^2}(-0.05)$$

$$= 0.05$$