# 27.6 Derivatives of Exponential Functions

Let $b$ be a positive constant.

$$\frac{d}{dx}\left[b^x\right] = (\ln b)\, b^x$$

Special Case: $\frac{d}{dx}\left[e^x\right] = e^x$

Ex: Find $f'(x)$

a) $f(x) = e^x$

$f'(x) = e^x$

b) $f(x) = 3^x$

$f'(x) = (\ln 3)\, 3^x$

Chain Rule for Exponential Functions

$$\frac{d}{dx}\left[b^u\right] = (\ln b)\, b^u \frac{du}{dx}$$

$b$: positive constant

$u$: function of $x$

Notation: $e^x$ is sometimes written $\exp(x)$

$e^{x^2}$ " $\exp(x^2)$

outside inside

**Ex:** Find $f'(x)$

a) $f(x) = e^{x^2}$

$f'(x) = e^{x^2}(2x)$

$= 2x e^{x^2}$

b) $f(x) = 2^{\sqrt{x}}$

$f'(x) = (\ln 2)\, 2^{\sqrt{x}} \frac{d}{dx}\left[x^{1/2}\right]$

$= (\ln 2)\, 2^{\sqrt{x}} \left(\frac{1}{2} x^{-1/2}\right)$ ✓

$= \frac{2^{\sqrt{x}} \ln 2}{2\sqrt{x}}$ ✓

c) $f(x) = e^{x+7}$

$f'(x) = e^{x+7}(1)$

$= e^{x+7}$

d) $f(x) = x^3 e^{\sqrt{x}}$

$(uv)' = uv' + vu'$

$f'(x) = x^3 \frac{d}{dx}\left[e^{\sqrt{x}}\right] + e^{\sqrt{x}}(3x^2)$

$= x^3\left[e^{\sqrt{x}} \frac{1}{2} x^{-1/2}\right] + 3x^2 e^{\sqrt{x}}$ ✓

$= \frac{1}{2} x^{5/2} e^{\sqrt{x}} + 3x^2 e^{\sqrt{x}}$ ✓

$= x^2 e^{\sqrt{x}} \left(\frac{1}{2}x^{1/2} + 3\right)$ ✓

## Exponent Rules

$$a^M \cdot a^N = a^{M+N}$$

$$\frac{a^M}{a^N} = a^{M-N}$$

$$(a^M)^N = a^{MN}$$

**Ex:** Find $f'(x)$

a) $f(x) = 9e^{2x}(e^{3x} + e^{4x})$

$f(x) = 9e^{5x} + 9e^{6x}$

$f'(x) = 9(e^{5x} \cdot 5) + 9(e^{6x} \cdot 6)$ ✓

$= 45e^{5x} + 54e^{6x}$ ✓

$= 9e^{5x}(5 + 6e^{x})$ ✓

b) $f(x) = \dfrac{e^{3x} + e^{4x}}{e^{2x}}$

$f(x) = e^{-2x}(e^{3x} + e^{4x})$

$$f(x) = e^x + e^{2x}$$

$$f'(x) = e^x + e^{2x}(2) \quad \checkmark$$

$$= e^x + 2e^{2x} \quad \checkmark$$

$$= e^x(1 + 2e^x) \quad \checkmark$$

c) $f(x) = (4e^{7x})^2 e^{6x}$

$$f(x) = 16e^{14x} \cdot e^{6x}$$

$$f(x) = 16e^{20x}$$

$$f'(x) = 16e^{20x}(20) \quad \checkmark$$

$$= 320e^{20x} \quad \checkmark$$

## 27.8 Applications of Ch 27

Ex: Approximate a solution to $x = \cos x$ in $0 \le x \le 1$.

Newton's Method

$\underbrace{x - \cos x = 0}_{f(x)}$

| $x$ | $f(x) = x - \cos x$ |
|---|---|
| 0 | $-1$ |
| 1 | $0.46$ ← |

Radian Mode

Choose $x_0 = 1$

$f'(x) = 1 + \sin x$

| $x_n$ | $f(x) = x - \cos x$ | $f'(x) = 1 + \sin x$ | $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ |
|---|---|---|---|
| 1 | 0.4597 | 1.8415 | 0.7504 (0.75) |
| 0.7504 | 0.0190 | 1.6819 | 0.7391 (0.74) |
| 0.7391 | 0.0000 | # | 0.7391 (0.74) |

Table to 4 decimal places
Answer to 2 "

$$\boxed{x \approx 0.74}$$