

Quiz Wed Oct 25 section 24.8
Wed Nov 1 27.3

Test 2

Thurs Oct 26

24.1-5, 24.7-8, 27.1-2

7 Questions

Bring: calculator, music/earplugs
Practice Problems on website

27.5 Review of Logarithms

3 Log Rules

$$\log_b(xy) = \log_b x + \log_b y$$

↑
any base

$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$$

$$\log_b x^n = n \log_b x$$

Ex: Simplify $\ln \sqrt{x}$

$$= \ln x^{1/2}$$
$$= \frac{1}{2} \ln x$$

Ex: Simplify $\ln \left[\frac{\sin(2x) \sqrt{x^2+6}}{(x+1)^4} \right]$

$$= \ln [\sin(2x) \sqrt{x^2+6}] - \ln (x+1)^4$$

$$= \ln [\sin(2x)] + \ln (x^2+6)^{1/2} - \ln (x+1)^4$$

$$= \ln [\sin(2x)] + \frac{1}{2} \ln (x^2+6) - 4 \ln (x+1)$$

Derivatives of Logarithmic Functions

$$\frac{d}{dx} [\log_b x] = \frac{1}{\ln b} \frac{1}{x} \quad (\star)$$

Special Case: $\frac{d}{dx} [\ln x] = \frac{1}{x} \quad (\star)$

Why? $\frac{d}{dx} [\ln x] = \frac{d}{dx} [\log_e x]$

$$= \frac{1}{\ln e} \frac{1}{x}$$
$$= \frac{1}{x}$$

Ex: Find $f'(x)$

a) $f(x) = \ln x$

$$f'(x) = \frac{1}{x}$$

b) $f(x) = \ln(4x^2 + 1)$

$$f'(x) = \frac{1}{4x^2 + 1} (8x) \quad \text{Chain Rule}$$

$$= \frac{8x}{4x^2 + 1}$$

c) $f(x) = \log_2 x$

$$f'(x) = \frac{1}{\ln 2} \cdot \frac{1}{x} \quad \checkmark$$

$$= \frac{1}{x \ln 2} \quad \checkmark$$

d) $f(x) = \log_3(7x + 1)$

$$f'(x) = \frac{1}{\ln 3} \cdot \frac{1}{7x + 1} (7)$$

$$= \frac{7}{(\ln 3)(7x + 1)}$$

$$e) f(x) = \log(x^2 + 2)$$

$$= \log_{10}(x^2 + 2)$$

$$f'(x) = \frac{1}{\ln 10} \frac{1}{x^2 + 2} (2x) \quad \checkmark$$

$$= \frac{2x}{(x^2 + 2) \ln 10} \quad \checkmark$$

$$f) f(x) = \ln[\sin x^2]$$

$$f'(x) = \frac{1}{\sin x^2} [\cos x^2 (2x)] \quad \checkmark$$

$$= \frac{2x \cos x^2}{\sin x^2} \quad \checkmark$$

$$= 2x \cot x^2 \quad \checkmark$$

Ex: $f(x) = \ln^4(x^3 + 17x + 1)$

Find $f'(x)$

$$f(x) = [\ln(x^3 + 17x + 1)]^4$$

Like $\sin^4 x = [\sin x]^4$

$$f'(x) = 4 \left[\ln(x^3 + 17x + 1) \right]^3 \left[\frac{1}{x^3 + 17x + 1} (3x^2 + 17) \right]$$
$$= 4 \ln^3(x^3 + 17x + 1) \frac{3x^2 + 17}{x^3 + 17x + 1}$$

Ex: $f(x) = \ln \frac{(4x+1)^2}{(5x+3)}$

Find $f'(x)$

Simplify $f(x)$ using log rules.

$$f(x) = \ln(4x+1)^2 - \ln(5x+3)$$
$$= 2 \ln(4x+1) - \ln(5x+3)$$

$$f'(x) = 2 \frac{1}{4x+1} (4) - \frac{1}{5x+3} (5) \quad \checkmark$$

$$= \frac{8}{4x+1} - \frac{5}{5x+3} \quad \checkmark$$

$$= \frac{8(5x+3) - 5(4x+1)}{(4x+1)(5x+3)} \quad \checkmark$$

$$= \frac{20x + 19}{(4x+1)(5x+3)} \quad \checkmark$$

27.6 Derivatives of Exponential Functions

logarithmic function $y = \log_2 x$

exponential function

$$y = 2^x$$

(variable is in the exponent)

Let b be a constant

$$\frac{d}{dx} [b^x] = (\ln b) b^x$$

SPECIAL CASE

$$\frac{d}{dx} [e^x] = e^x$$

Why? $\frac{d}{dx} [e^x] = (\ln e) e^x = e^x$