27.3 Derivatives of Inverse Trig Functions Cont'd

$f(x)$	$f'(x)$
$\sin^{-1} x$	$\dfrac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$\dfrac{-1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\dfrac{1}{1+x^2}$

Ex: Find $f'(x)$

e) $f(x) = \cos^{-1} \sqrt[3]{x}$

$$f'(x) = \frac{-1}{\sqrt{1-(x^{1/3})^2}} \left(\frac{1}{3} x^{-2/3}\right)$$

$$= \frac{-1}{3x^{2/3}\sqrt{1-x^{2/3}}}$$

f) $f(x) = \left[\tan^{-1} 8x - 5x^2\right]^3$

$$f'(x) = 3\left[\tan^{-1} 8x - 5x^2\right]^2 \left(\frac{1}{1+(8x)^2}(8) - 10x\right)$$

$$= 3\left[\tan^{-1} 8x - 5x^2\right]^2 \left(\frac{8}{1+64x^2} - 10x\right)$$

or $$3\left[\tan^{-1} 8x - 5x^2\right]^2 \frac{8 - 10x - 640x^3}{1+64x^2}$$

Ex: $$y = \frac{\tan^{-1} x}{1+x^2}$$

Find $\left.\frac{dy}{dx}\right|_{x=-1}$

$$\frac{dy}{dx} = \frac{(1+x^2)\frac{1}{1+x^2} - (\tan^{-1} x)(2x)}{(1+x^2)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=-1} = \frac{1 + 2\tan^{-1}(-1)}{4}$$

$\tan^{-1} 1 = \frac{\pi}{4}$

$\tan^{-1}(-1) = -\frac{\pi}{4}$

$$= \frac{1 + 2\left(-\frac{\pi}{4}\right)}{4} \cdot \frac{2}{2}$$

$$= \frac{2 - \pi}{8}$$

27.5 Review: Logarithms

$\log_2 1 = 0 \qquad 2^{?} = 1$

$\log_2 2 = 1$

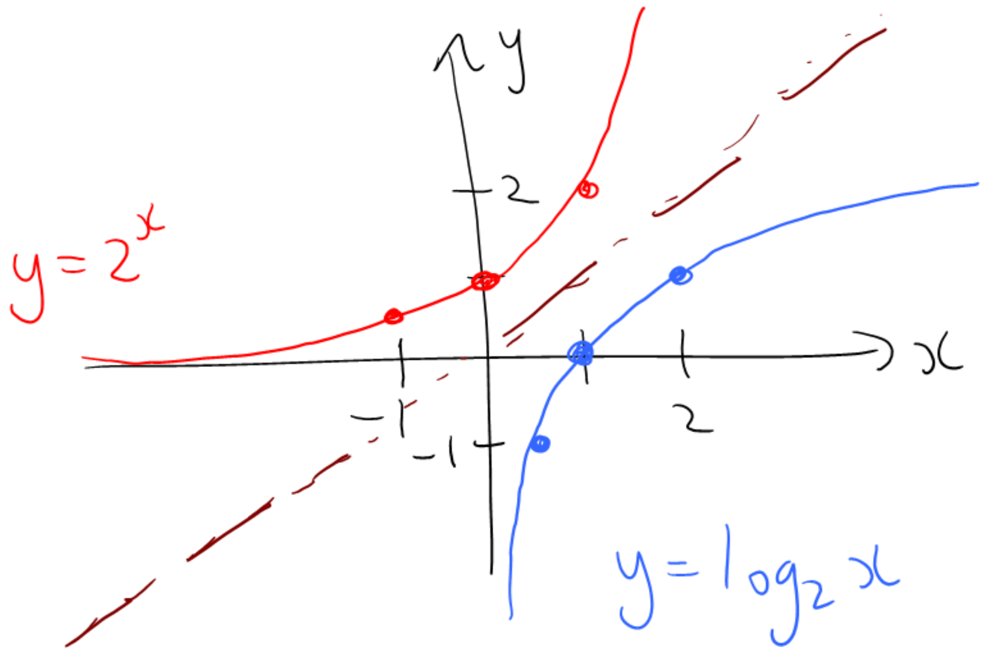
$\log_2 8 = 3$

$\log_2 \frac{1}{2} = -1 \qquad 2^{?} = \frac{1}{2}$

$\log_2 2^7 = 7$

x	$y=2^x$
-1	$\frac{1}{2}$
0	1
1	2

x	$y=\log_2 x$
$\frac{1}{2}$	-1
1	0
2	1



Euler's Number $e \approx 2.718$

<u>Notation</u>: $\log x$ means $\log_{10} x$

$\ln x$ means $\log_e x$

pronounced "lawn x"

Ex:

$$\log 10 = \log_{10} 10$$
$$= 1$$

$$\log 0.1 = \log_{10} 0.1$$
$$= -1$$

$$\ln e = \log_e e$$
$$= 1$$

$$\ln e^4 = 4$$

Change of Base Formula

$$\log_b a = \frac{\ln a}{\ln b}$$

Ex: Find $\log_2 5$

$$= \frac{\ln 5}{\ln 2}$$
$$\approx 2.32$$

<u>Ex</u>: Simplify

a) $\log e$

$$= \frac{\ln e}{\ln 10}$$

$$= \frac{1}{\ln 10}$$

$\ln e = 1$

b) $\log_b e$

$$= \frac{\ln e}{\ln b}$$

$$= \frac{1}{\ln b}$$