27.3 Review: Inverse Trig Functions

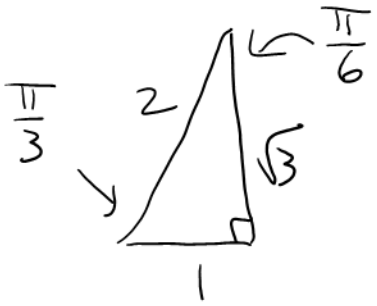
$$\sin(\text{angle}) = \#$$

$$\arcsin(\#) = \text{angle}$$

$$\arcsin x = \sin^{-1} x$$

NOT TO BE CONFUSED WITH

$$(\sin x)^{-1} = \frac{1}{\sin x} = \csc x$$



$$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\sin^{-1} \frac{\sqrt{3}}{2} = \frac{\pi}{3}$$

$$\arcsin \frac{1}{2} = \frac{\pi}{6}$$

$$\boxed{\sin^{-1}(-a) = -\sin^{-1} a}$$

$$\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$$

$$\arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$$

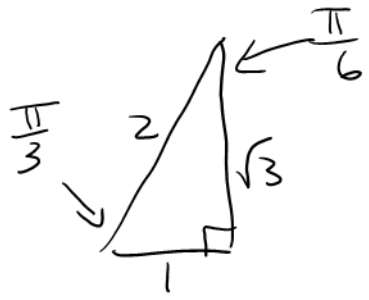
$\tan(\text{angle}) = \#$

$\arctan(\#) = \text{angle}$

$\arctan x = \tan^{-1} x$

NOT TO BE CONFUSED WITH

$(\tan x)^{-1} = \frac{1}{\tan x} = \cot x$



$\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$

$\arctan(\sqrt{3}) = \frac{\pi}{3}$

$\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

$\tan\left(\frac{\pi}{4}\right) = 1$

$\tan^{-1} 1 = \frac{\pi}{4}$

$$\arctan(-a) = -\arctan a$$

$\tan^{-1}(-1) = -\frac{\pi}{4}$

$\arctan(-\sqrt{3}) = -\frac{\pi}{3}$

Ex: $f(x) = \sqrt{4 - \sin^{-1}(3x-8)}$

Find $f(2.8)$

$f(2.8) = \sqrt{4 - \sin^{-1}(0.4)}$

($\sin^{-1}(0.4)$: angle)

Radian Mode

≈ 1.9

Ex: $f(x) = \cos^{-1}\left(\frac{\cos 2x}{x+1}\right)$

Find $f\left(\frac{\pi}{7}\right)$

Radian Mode

$f\left(\frac{\pi}{7}\right) = \cos^{-1}\left(\frac{\cos\left(\frac{2\pi}{7}\right)}{\left(\frac{\pi}{7}+1\right)}\right)$

≈ 1.1 radians

27.3 Derivatives of Inverse Trig Functions

$$\frac{d}{dx}[\arcsin x] = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}[\arccos x] = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}[\arctan x] = \frac{1}{1+x^2}$$

Ex: Find $f'(x)$

a) $f(x) = \sin^{-1}(7x)$

$$f'(x) = \frac{1}{\sqrt{1-(7x)^2}}(7)$$

$$= \frac{7}{\sqrt{1-49x^2}}$$

b) $f(x) = \frac{1}{3}\sin^{-1}\left(\frac{x}{4}\right)$

$$f'(x) = \frac{1}{3}\frac{1}{\sqrt{1-\left(\frac{x}{4}\right)^2}}\left(\frac{1}{4}\right)$$

$$= \frac{1}{3\cdot\sqrt{1-\frac{x^2}{16}}\sqrt{16}}$$

$$= \frac{1}{3 \cdot \sqrt{16 - x^2}}$$

$\sqrt{a}\sqrt{b} = \sqrt{ab}$

$\sqrt{9}\sqrt{1 - \frac{x^2}{9}} = \sqrt{9 - x^2}$

c) $f(x) = x \cos^{-1} x^2$

$$f'(x) = x\left[\frac{-1}{\sqrt{1-(x^2)^2}}(2x)\right] + \left(\cos^{-1} x^2\right)(1)$$

$$= \frac{-2x^2}{\sqrt{1-x^4}} + \cos^{-1} x^2$$

d) $f(x) = \tan^{-1} 2x$

$$f'(x) = \frac{1}{1+(2x)^2}\ (2)$$

$$= \frac{2}{1+4x^2}$$