

Quiz Wed Oct 18 Section 24.7

Wed Oct 25 Section 24.8

Test Thurs Oct 26

24.1-24.5, 24.7-24.8, 27.1-27.2

27.1 Derivatives of Sine and Cosine

$$\frac{d}{dx} [\sin x] = \cos x$$

$$\frac{d}{dx} [\cos x] = -\sin x$$

Ex: a) $\frac{d}{dx} \sin x^2$

$$= (\cos x^2)(2x)$$
$$= 2x \cos x^2$$

b) $\frac{d}{dx} \sin^2 x$

$$= \frac{d}{dx} [\sin x]^2$$
$$= 2 [\sin x] (\cos x)$$
$$= 2 \sin x \cos x$$

Chain Rule $\frac{d}{dx} (1+4x)^2 = 2(1+4x)(4)$

Notation $\cos^3 6x = [\cos 6x]^3$

Ex: Find $f'(x)$

a) $f(x) = 4 \sin(7x+2)$

$$f'(x) = 4 \cos(7x+2) (7)$$
$$= 28 \cos(7x+2)$$

b) $f(x) = \cos^4 x$

$$f(x) = [\cos x]^4$$

$$f'(x) = 4 [\cos x]^3 (-\sin x)$$

$$= -4 \cos^3 x \sin x$$

$$\text{or } -4 \sin x \cos^3 x$$

c) $f(x) = \cos^2 \left(1 - \frac{\pi x}{2}\right)$

$$f(x) = \left[\cos \left(1 - \frac{\pi x}{2}\right) \right]^2$$

$$f'(x) = 2 \left[\cos \left(1 - \frac{\pi x}{2} \right) \right] \left[-\sin \left(1 - \frac{\pi x}{2} \right) \left(-\frac{\pi}{2} \right) \right]$$

$$= \pi \cos \left(1 - \frac{\pi x}{2} \right) \sin \left(1 - \frac{\pi x}{2} \right)$$

d) $f(x) = x \cos 7x$

$$f'(x) = x \frac{d}{dx} [\cos 7x] + (\cos 7x) (1)$$

$$(uv)' = uv' + vu'$$

$$= x [-\sin 7x \cdot 7] + \cos 7x$$

$$= -7x \sin 7x + \cos 7x$$

e) $f(x) = \frac{\sin x}{1+x^2}$

$$\left(\frac{u}{v} \right)' = \frac{v u' - u v'}{v^2}$$

$$f'(x) = \frac{(1+x^2) \cos x - \sin x (2x)}{(1+x^2)^2}$$

$$= \frac{(1+x^2) \cos x - 2x \sin x}{(1+x^2)^2}$$

Recall Implicit Differentiation (Section 23.8)

$$\frac{d}{dx} [x^3] = 3x^2$$

$$\frac{d}{dx} [y^3] = 3y^2 \frac{dy}{dx}$$

Ex: Find $\frac{dy}{dx}$ for

$$\sin(xy) + \cos 3y = x^3$$

Take $\frac{d}{dx}$:

$$\underline{\underline{\cos(xy) [x \frac{dy}{dx} + y(1)] - \sin 3y (3 \frac{dy}{dx}) = 3x^2}}$$

Solve for $\frac{dy}{dx}$:

$$x \cos(xy) \frac{dy}{dx} + y \cos xy - 3 \sin(3y) \frac{dy}{dx} = 3x^2$$

$$[x \cos xy] \frac{dy}{dx} - [3 \sin 3y] \frac{dy}{dx} = 3x^2 - y \cos xy$$

$$[x \cos xy - 3 \sin 3y] \frac{dy}{dx} = 3x^2 - y \cos xy$$

$$\frac{dy}{dx} = \frac{3x^2 - y \cos xy}{x \cos xy - 3 \sin 3y}$$

27.2 Derivatives of other Trig Functions

$$\frac{d}{dx} [\tan x] = \sec^2 x$$

$$\frac{d}{dx} [\sec x] = \sec x \tan x$$

$$\frac{d}{dx} [\csc x] = -\csc x \cot x$$

$$\frac{d}{dx} [\cot x] = -\csc^2 x$$

$$\frac{d}{dx} [\sin x] = \cos x$$

$$\frac{d}{dx} [\cos x] = -\sin x$$

Ex:

$f(x)$	$f'(x)$
$\sin x^3$	$3x^2 \cos x^3$
$\cos 8x$	$-8 \sin 8x$
$\tan 7x$	$7 \sec^2 7x$
$\sec x^9$	$9x^8 \sec x^9 \tan x^9$
$\sec(2x)$	$2 \sec(2x) \tan(2x)$
$\csc(6x+1)$	$-6 \csc(6x+1) \cot(6x+1)$
$\csc x^6$	$-6x^5 \csc x^6 \cot x^6$
$\cot x^2$	$-2x \csc^2 x^2$