Quiz Wed Oct 18 24.7

24.8 Differentials and Linear Approximation

Ex:

$$y = 3x^3$$

$$\frac{dy}{dx} = 9x^2$$

$$dy = 9x^2\,dx$$

dy ← differential of y

dx ← differential of x

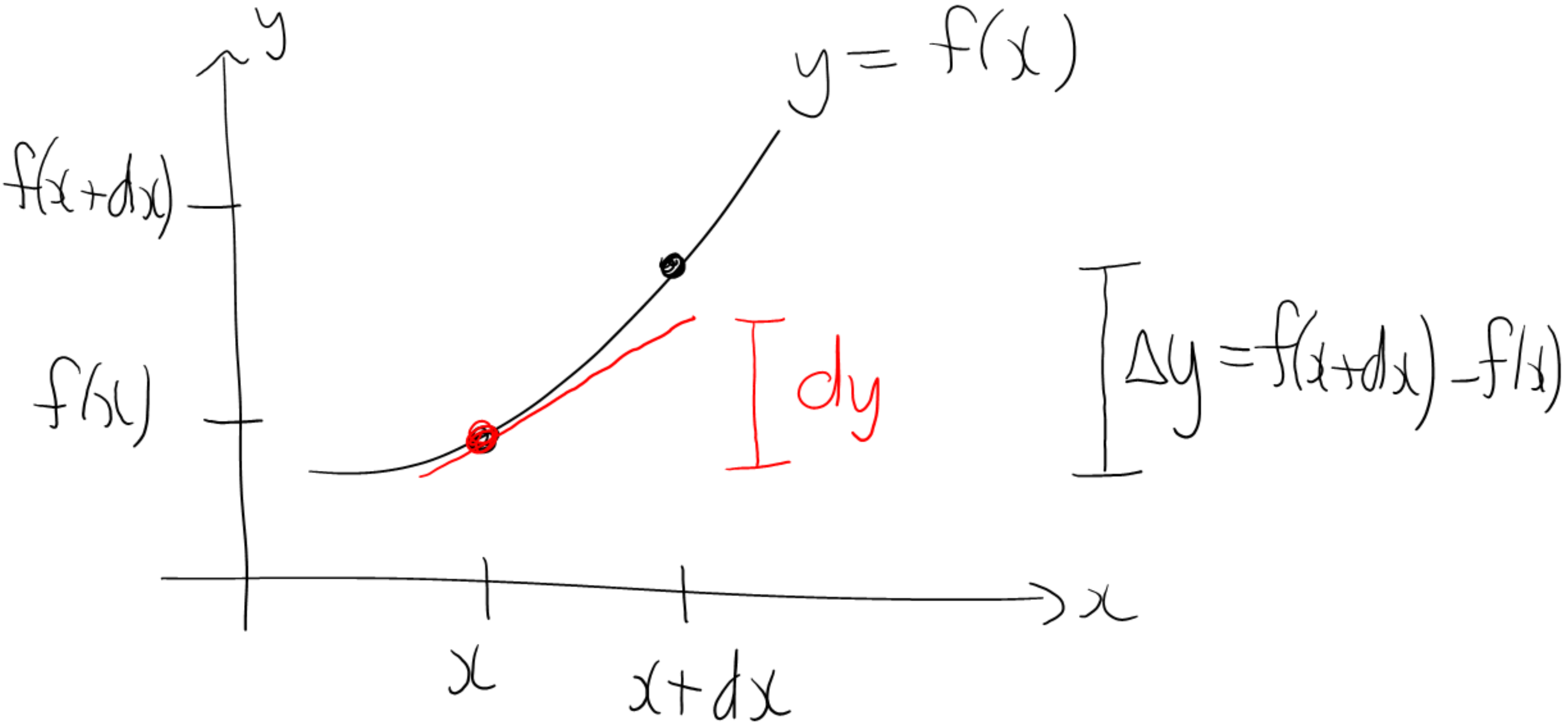
Ex:

$$V = \frac{4}{3}\pi r^3$$

Find the differential of V

$$\frac{dV}{dr} = 4\pi r^2$$

$$dV = 4\pi r^2\,dr$$



dx = small change in x

dy = rise in tangent line

Δy = true change in y

FACT

When dx is small,

$$\Delta y \approx dy$$

hard to calculate → Δy; dy ← easy to calculate

Notation: Say $y = x^2$

x is the independent variable

dx and Δx mean the same thing

y is the dependent variable

dy and Δy are different

Ex: $f(x) = x^2 + 2$

Compute Δy and dy

for $x = 1$ and $dx = 0.05$

$$\begin{aligned}\Delta y &= f(x + dx) - f(x) \\ &= f(1.05) - f(1) \\ &= 3.1025 - 3 \\ &= 0.1025\end{aligned}$$

$$\frac{dy}{dx} = 2x$$

$$dy = 2x\,dx$$
$$= 2(1)(0.05)$$
$$= 0.1$$

Ex: Approximate Δy using dy

$y = x^{1/2}$ $x = 4$ $dx = 0.1$

$$\frac{dy}{dx} = \frac{1}{2}x^{-1/2}$$

$$dy = \frac{1}{2}x^{-1/2}\,dx$$

$$\Delta y \approx dy$$
$$\approx \frac{1}{2}x^{-1/2}\,dx$$
$$\approx \frac{1}{2}(4)^{-1/2}(0.1)$$
$$\approx 0.025$$

Absolute Error = measured − true

$$\text{Relative Error} = \frac{\text{measured} - \text{true}}{\text{true}}$$

Ex: Sphere's radius is measured to be 2.500 cm. If the true radius is 2.512 cm, find the absolute error and the relative error.

absolute error = measured - true

$= 2.500 - 2.512$

$= -0.012$ cm ← dr

relative error = $\dfrac{\text{measured} - \text{true}}{\text{true}}$

$= \dfrac{2.500 - 2.512}{2.512}$

≈ -0.0048

or -0.48% ← $\dfrac{dr}{r}$

Why Use Differentials?

Absolute error in radius $= dr$

Relative " $= \frac{dr}{r}$

Relative error in volume of sphere

$$= \frac{dV}{V}$$

$$= \frac{4\pi r^2\, dr}{\left(\frac{4}{3}\pi r^3\right)}$$

$$= \frac{3r^2 dr}{r^3}$$

$$= \frac{3\,dr}{r}$$

$$= 3\left(\frac{dr}{r}\right)$$

← relative error in radius

← dimension of object

$\frac{\Delta V}{V}$ and $\frac{\Delta r}{r}$ do not have a predictable relationship.

Preview:

Area of a circle $A = \pi r^2$

$$\frac{dA}{A} = 2\frac{dr}{r}$$