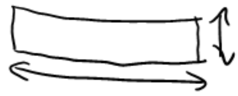
24.7 Cont'd

See handout.

6

$V_{cylinder} = \pi r^2 h$

$SA_{cylinder} = \underbrace{2\pi r^2}_{\text{top and bottom}} + \underbrace{2\pi r h}_{\text{side}}$



1) Given Info

Restriction: $\pi r^2 h = 100$

Minimize $f = 2\pi r^2 + 2\pi r h$

2) Single-Variable Function

$$\pi r^2 h = 100$$

$$h = \frac{100}{\pi r^2} \rightarrow f = 2\pi r^2 + 2\pi r h$$

$$f = 2\pi r^2 + 2\pi r\left(\frac{100}{\pi r^2}\right)$$

$$f = 2\pi r^2 + 200 r^{-1}$$

3) Set $f' = 0$

$$f' = 4\pi r - 200 r^{-2}$$

Set $f' = 0$: $0 = 4\pi r - 200 r^{-2}$

$$\frac{200}{r^2} = 4\pi r$$

$$200 = 4\pi r^3$$

$$\frac{200}{4\pi} = r^3$$

$$\frac{50}{\pi} = r^3$$

$$\sqrt[3]{\frac{50}{\pi}} = r$$

4) Answer

$$r = \sqrt[3]{\frac{50}{\pi}} \quad \text{or} \quad 2.52 \text{ cm}$$

$$h = \frac{100}{\pi r^2}$$

$$= \frac{100}{\pi\left(\frac{50}{\pi}\right)^{2/3}} \quad \text{or} \quad 5.03 \text{ cm}$$

24.8 Differentials and Linear Approximation

Application : Relative Error
Absolute Error

Ex:

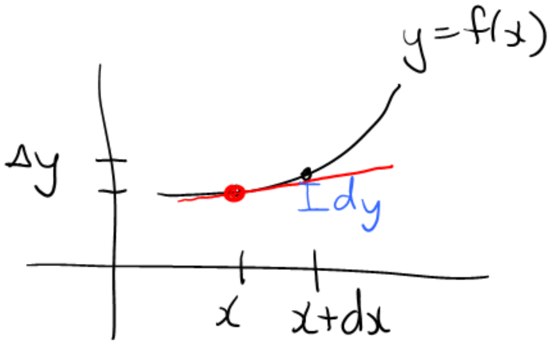
$$y = 3x^6 + 7x^3$$

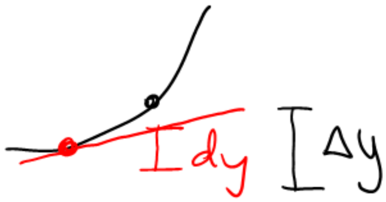
$$\frac{dy}{dx} = 18x^5 + 21x^2$$

differential of y

$$\rightarrow dy = (18x^5 + 21x^2)\,dx \leftarrow$$

differential of x





dx: small change in x

dy: rise of tangent line

Δy: change in y

FACT $\quad \Delta y \approx dy$ when dx is small