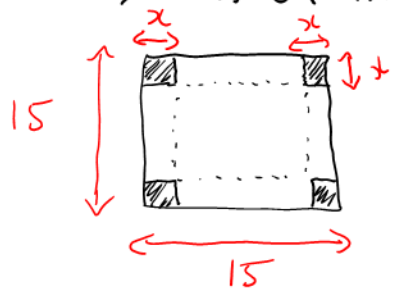
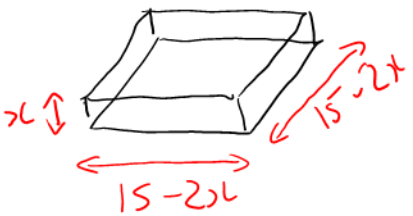
24.7 Cont'd
See handout on website.

(2)

1) Given info





Maximize $V = lwh$

$= (15-2x)^2 x$

2) Single-variable function

$f = (15-2x)^2 x$

3) Set $f' = 0$

$f' = (15-2x)^2(1) + x[2(15-2x)(-2)]$

$= (15-2x)(15-2x - 4x)$

$= (15-2x)(15-6x)$

Set $f' = 0$: $(15-2x)(15-6x) = 0$

$15-2x = 0$
$15 = 2x$
$x = 7.5$
$l = w = 0$
$V = 0$
REL MIN

$15-6x = 0$
$15 = 6x$
$x = 2.5$
REL MAX

4) Answer

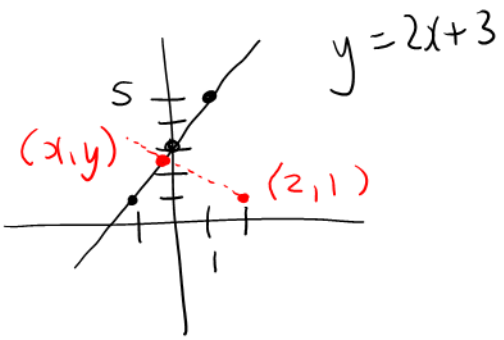
The height is $x = 2.5$ cm

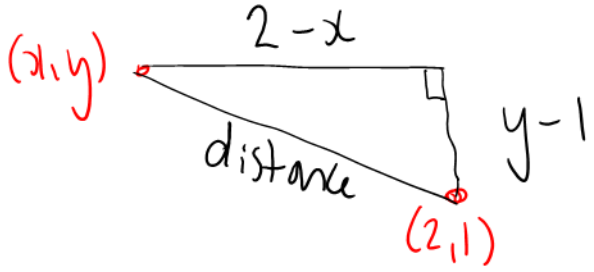
The max volume is $[15-2(2.5)]^2(2.5) = 250\text{ cm}^3$

OMIT ③

④

1) Given info





Minimize distance $= \sqrt{(2-x)^2 + (y-1)^2}$

Restriction: $y = 2x+3$

2) Single-variable function

$$f = \sqrt{(2-x)^2 + (2x+3-1)^2}$$

$$= \sqrt{(2-x)^2 + (2x+2)^2}$$

3) Set $f' = 0$

$$f' = \frac{1}{2}\left[(2-x)^2+(2x+2)^2\right]^{-1/2}\left[2(2-x)(-1)+2(2x+2)(2)\right]$$

Set $f' = 0$:

$$\frac{(4-2x)(-1)+(4x+4)(2)}{2\sqrt{\quad}} = 0$$

Recall: $\frac{a}{b} = 0 \Rightarrow a = 0$

$$(4-2x)(-1)+(4x+4)(2)=0$$
$$-4+2x+8x+8=0$$
$$10x+4=0$$
$$10x=-4$$
$$x=-0.4$$

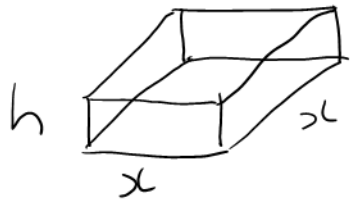
4) Answer

$x=-0.4 \longrightarrow y=2x+3$

$y=2.2$

The point is $(-0.4, 2.2)$

(5) 1) Given info



Top and Sides : $\$1/cm^2$

Base : $\$2/cm^2$

Maximize $V=x^2h$

Cost = \$144

Restriction: $144 = \underbrace{2x^2}_{\text{BASE}} + \underbrace{4(1)xh}_{\text{4 SIDES}} + \underbrace{1(x^2)}_{\text{TOP}}$

\$; $\frac{\$}{cm^2}$; cm^2

$$3x^2 + 4xh = 144$$

2) Single-Variable Function

$$3x^2 + 4xh = 144$$

$$4xh = 144 - 3x^2$$

$$h = \frac{144 - 3x^2}{4x} \longrightarrow V = x^2 h$$

$$V = \frac{x^2(144 - 3x^2)}{4x}$$

$$V = \frac{144x - 3x^3}{4}$$

$$V = \frac{1}{4}(144x - 3x^3)$$

3) Set $V' = 0$

$$V' = \frac{1}{4}(144 - 9x^2)$$

Set $V' = 0$:

$$0 = \frac{1}{4}(144 - 9x^2)$$

$$0 = 144 - 9x^2$$

$$9x^2 = 144$$

$$x^2 = 16$$

$$x = \pm 4$$

$$x = 4$$

4) Answer

$x = 4 \longrightarrow h = \dfrac{144 - 3x^2}{4x}$

$= \dfrac{144 - 48}{16}$

$= 6$

The dimensions are $4\text{cm} \times 4\text{cm} \times 6\text{cm}$.