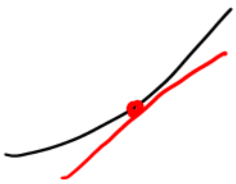
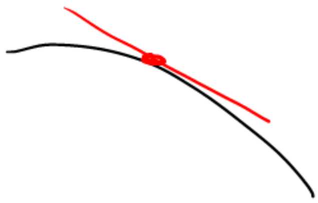
24.5 Sketching Polynomials

Properties of polynomials will be useful in Section 24.7



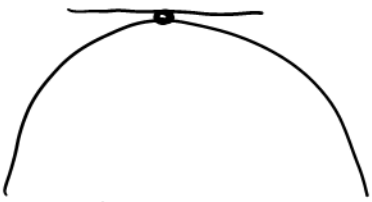
$f(x)$ is increasing

$f'(x) > 0$

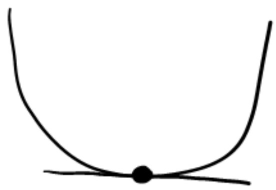


$f(x)$ is decreasing

$f'(x) < 0$



relative maximum

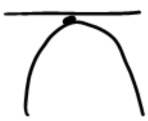


relative minimum

Def

x-values where $f'(x) = 0$ are called <u>critical points</u>. They are <u>possible</u> relative maximum or relative minimum points.

Critical Points:

 or  or  or 

<u>Ex</u>: $f(x) = 2x^3 - 9x^2 - 240x + 5$

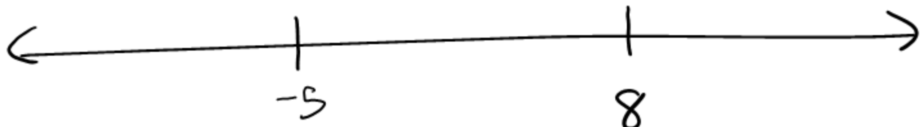
Find all relative maximum and relative minimum points.

$f'(x) = 6x^2 - 18x - 240$

Set $f'(x) = 0$:

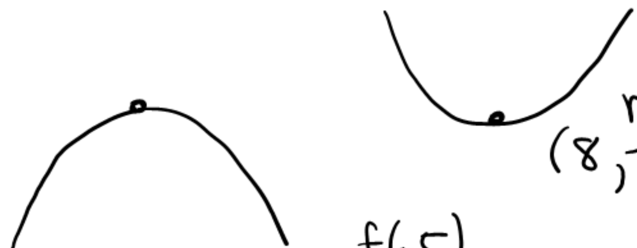
$$6x^2 - 18x - 240 = 0$$
$$6(x^2 - 3x - 40) = 0$$
$$6(x-8)(x+5) = 0$$
$$x = 8, -5$$

These are the only x-values where $f'(x)$ can change sign.
Sign change is not guaranteed.



	$x < -5$	$-5 < x < 8$	$x > 8$
$f'(x)$	$+$	$-$	$+$
$f(x)$	INCREASING	DEC	INC

Test a value in each interval.

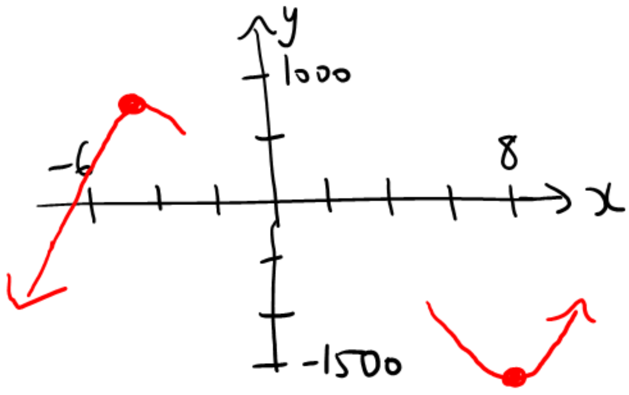


rel. min. $(8, -1467)$ ← $f(8)$

rel. max. $(-5, 730)$ ← $f(-5)$

Follow-Up:

Rough graph of $y = f(x)$



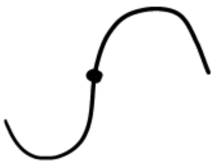
Concave up

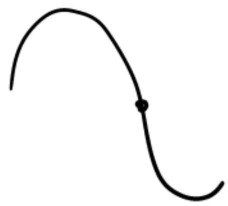
$f''(x) > 0$

Concave down

$f''(x) < 0$

<u>Point of Inflection</u>:





Points where $f''(x) = 0$ are <u>possible</u> points of inflection.

<u>Ex</u>: $f(x) = 2x^3 - 9x^2 - 240x + 5$

Find all points of inflection.

$f'(x) = 6x^2 - 18x - 240$

$f''(x) = 12x - 18$

Set $f''(x) = 0$: $\quad 12x - 18 = 0$

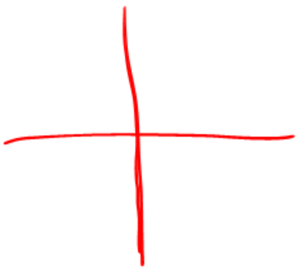
$$12x = 18$$
$$x = 1.5$$

		1.5	
$f''(x)$	$\ominus$		$\oplus$
$f(x)$	C.D.		C.U.

Point of Inflection $(1.5, -368.5)$ ← $f(1.5)$

Ex: Same $f(x)$

Find y-intercept and graph.



$x=0$

$x = 0 \rightarrow f(x) = 2x^3 - 9x^2 - 240x + 5$

$f(0) = 5$

$(0, 5)$