

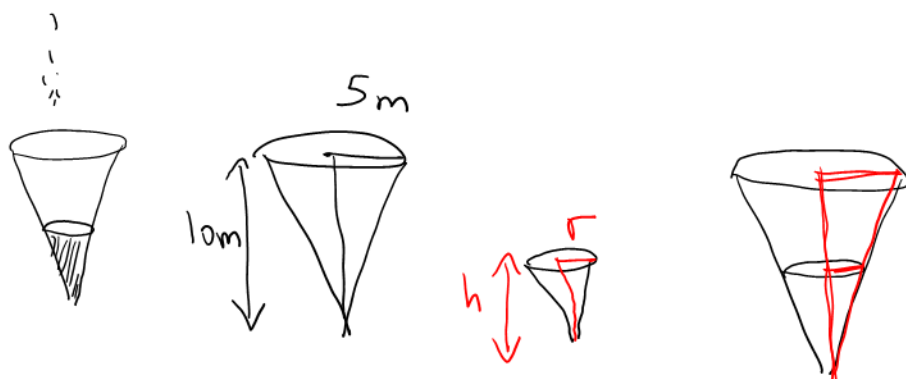
Quiz Wed Oct 11 Section 24.4

Final Exam Period

Mon Dec 11 - Tues Dec 19 (includes Sat Dec 16)

24.4 Related Rates

④



$$\frac{dV}{dt} = 10 \frac{\text{m}^3}{\text{s}}$$

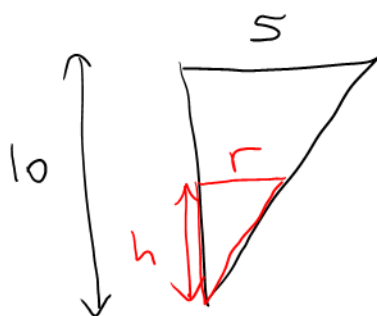
$$\frac{dr}{dt} = ?$$

$$r = 2 \text{ m}$$

1) Equation

$$V = \frac{\pi}{3} r^2 h$$

Eliminate h: Similar triangles



$$\frac{h}{r} = \frac{10}{5}$$

$$h = 2r$$

$$V = \frac{\pi}{3} r^2 (2r)$$

$$V = \frac{2\pi}{3} r^3$$

2) Take $\frac{d}{dt}$

$$\frac{dV}{dt} = \frac{dV}{dr} \frac{dr}{dt}$$

$$\frac{dV}{dt} = 2\pi r^2 \frac{dr}{dt}$$

3) Missing values?
NONE

4) Solve

$$10 = 2\pi (2)^2 \frac{dr}{dt}$$

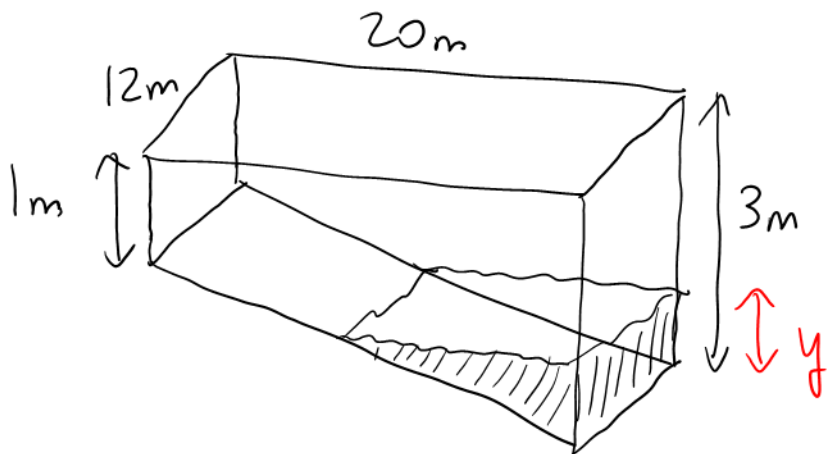
$$\frac{10}{8\pi} = \frac{dr}{dt}$$

$$\frac{dr}{dt} \approx 0.4 \frac{m}{s}$$

Skip (5) from handout

(6) A $12m \times 20m$ swimming pool is filled at a rate of $4 \text{ m}^3/\text{min}$. Ends are $1m$ deep and $3m$ deep, with a constant slope along the bottom.

Rate of change of water depth
when depth = 1.5m ?

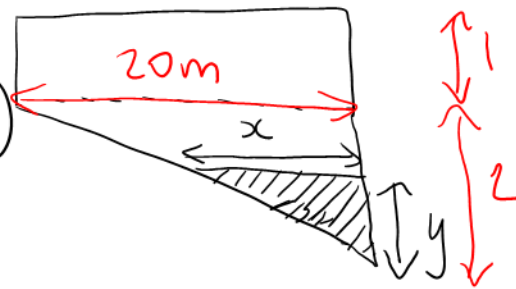


$$\frac{dV}{dt} = 4 \frac{\text{m}^3}{\text{min}}$$

$$\frac{dy}{dt} = ? \quad y = 1.5\text{m}$$

1) Equation

$$\begin{aligned} V &= (\text{triangular area})(\text{width of pool}) \\ &= \frac{1}{2}xy(12) \\ &= 6xy \end{aligned}$$



Eliminate x: Similar triangles

$$\frac{x}{y} = \frac{20}{2}$$

$$x = 10y$$

$$\begin{aligned} V &= 6(10y)y \\ V &= 60y^2 \end{aligned}$$

2) Take $\frac{d}{dt}$

$$\frac{dV}{dt} = \frac{dV}{dy} \frac{dy}{dt}$$

$$\frac{dV}{dt} = 120y \frac{dy}{dt}$$

~~3) Missing Values~~

4) Solve

$$4 = 120(1.5) \frac{dy}{dt}$$

$$\frac{dy}{dt} = \frac{4}{180} \frac{m}{min}$$

$$\text{or } 0.02 \frac{m}{min}$$