Quiz Wed Oct 4 Section 24.2
Wed Oct 11 Section 24.4

24.4 Related Rates

Variables x, y, r etc. all depend on time.

Ex: $V = \frac{4}{3}\pi r^3$

r depends on t

Find $\frac{dV}{dt}$

$$\frac{dV}{dt} = \frac{dV}{dr}\frac{dr}{dt}$$ Chain Rule

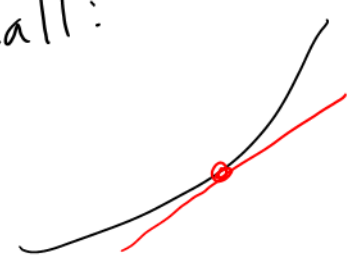
$$= 4\pi r^2 \frac{dr}{dt}$$

Ex: x and y depend on t

Find $\frac{d}{dt}[x^2 + y^2]$

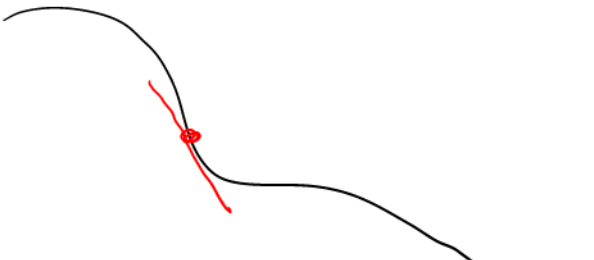
$$= 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$ Chain Rule

Recall:



$f(x)$ is increasing

$f'(x) > 0$

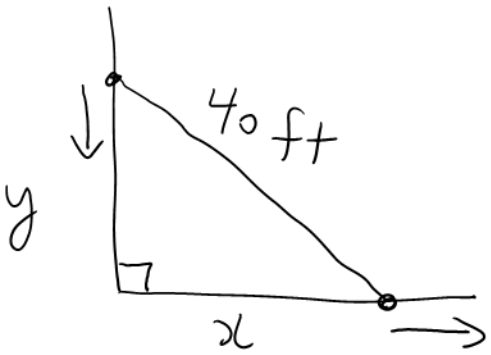


$f(x)$ is decreasing

$f'(x) < 0$

SEE HANDOUT ON WEBSITE

(1)



$\frac{dx}{dt} = 2 \frac{ft}{min}$

(positive because x is increasing)

$\frac{dy}{dt} = ?$

$x = 24$

1) Equation

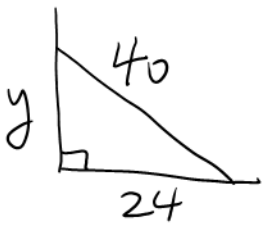
$$x^2 + y^2 = 40^2$$

2) Take $\frac{d}{dt}$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

3) Missing Values?

y is missing



$$24^2 + y^2 = 40^2$$

$$y^2 = 1024$$

~~$y = \pm 32$~~ $y = 32$

4) Solve

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$2(24)(2) + 2(32)\frac{dy}{dt} = 0$$

$$64\frac{dy}{dt} = -96$$

$$\frac{dy}{dt} = -1.5 \ \frac{ft}{min}$$

(2) $\frac{dr}{dt} = 3 \ \frac{cm}{s}$

$\frac{dV}{dt} = ?$

$SA = 100\pi \ cm^2$

$V_{sphere} = \frac{4}{3}\pi r^3$

$SA_{sphere} = 4\pi r^2$

1) Equation

$$V = \frac{4}{3}\pi r^3$$

2) Take $\frac{d}{dt}$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

3) Missing Values?

$$SA = 100\pi$$
$$4\pi r^2 = 100\pi$$

4) Solve

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$= 100\pi(3)$$
$$= 300\pi \ \frac{cm^3}{s}$$

Know all formulas from class: V, SA of sphere etc.

(3) y' = slope of tangent line

$\frac{dy'}{dt} = ?$

$y = 4(2+5x)^{-1}$

$x = 2$

$\frac{dx}{dt} = 0.5 \ \frac{\text{units}}{s}$

1) Equation

$y' = -4(2+5x)^{-2}(5)$

$y' = -20(2+5x)^{-2}$

2) Take $\frac{d}{dt}$: $\frac{dy'}{dt} = \frac{dy'}{dx}\frac{dx}{dt}$

$$\frac{dy}{dt} = 40(2+5x)^{-3}(5)\ \frac{dx}{dt}$$

3) Missing Values?

NONE

4) Solve

$$\frac{dy}{dt} = 40(12)^{-3}(5)(0.5)$$

$$\approx 0.06\ \frac{\text{units}}{s}$$