Quiz Wed Oct 4 Section 24.2

24.3 Curvilinear Motion

Ex: $x = 3t^2$ $y = 1 - t^2$

Find velocity at $t = 4$ s.

$v_x = 6t$ $v_y = -2t$

@ $t=4$: $v_x = 24$ $v_y = -8$

speed $v = \sqrt{v_x^2 + v_y^2}$

$= \sqrt{24^2 + (-8)^2}$

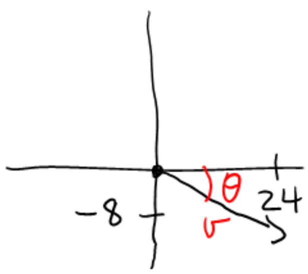
≈ 25.3 m/s

direction $\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$ $(+180°?)$

$\boxed{\text{Add } 180° \text{ when } v_x < 0}$

$= \tan^{-1}\left(\frac{-8}{24}\right)$

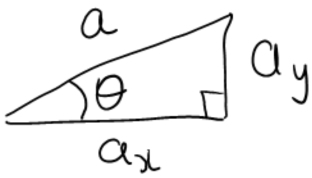
$\approx -18.4°$



Acceleration in x-direction $a_x = \frac{d}{dt}[v_x]$

" y-direction $a_y = \frac{d}{dt}[v_y]$

Acceleration has magnitude and direction.



Magnitude of acceleration $a = \sqrt{a_x^2 + a_y^2}$

Direction " $\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right)$ (+180°?)

(Add 180° when $a_x < 0$)

Ex: $x = 5 + t^2$ $y = 1 + t^3$

Find acceleration at $t = 3$ s.

$v_x = 2t$ $v_y = 3t^2$

$a_x = 2$ $a_y = 6t$

@ $t=3$: $a_x = 2$ $a_y = 18$

magnitude $a = \sqrt{2^2 + 18^2}$

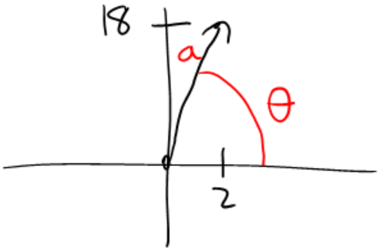
≈ 18.1 m/s²

direction $\theta = \tan^{-1}\left(\frac{18}{2}\right)$ (+180°?)

(Add 180° when $a_x < 0$)

$= \tan^{-1}\left(\frac{18}{2}\right)$

$\approx 83.7°$



Recap of Chain Rule

Ex: $y = 100 - 2x + 8x^2$

and x depends on t.

Find $\frac{dy}{dt}$.

Chain Rule (Formal Version)

$$\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt}$$

$$= (-2 + 16x)\frac{dx}{dt}$$

Quick Ex: $y = 7x^3 - 2x + 1$

x depends on t

Find $\frac{dy}{dt}$

$$\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt}$$

$$= (21x^2 - 2)\frac{dx}{dt}$$

Ex: Given position curve $y = 100 - 0.02x^2$

and $V_x = 9$ m/s (constant).

Find velocity at $t = 3$ s.

$$\frac{dy}{dt} = \frac{dy}{dx}\,\frac{dx}{dt}$$

v_y v_x

$$v_y = (-0.04x)\,v_x$$

$x = ?$

$x =$ (Initial Position) + (Speed)(Time)

0 unless otherwise specified

$$x = 0 + 9(3)$$
$$= 27$$

$$v_y = -0.04(27)(9)$$
$$= -9.72$$

speed $\quad v = \sqrt{9^2 + (-9.72)^2} \approx 13.2\ \frac{m}{s}$

direction $\quad \theta = \tan^{-1}\left(\frac{-9.72}{9}\right)\ (+180°?)$

$$= \tan^{-1}\left(\frac{-9.72}{9}\right)$$

$$\approx -47.2°$$