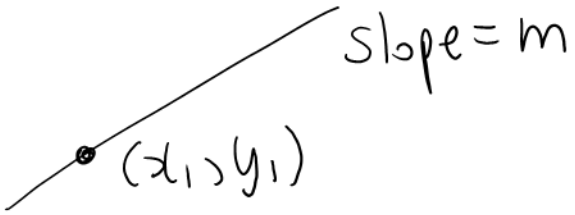
# 24.1 Tangents and Normals

## Point-Slope Equation of a Line




$$y - y_1 = m(x - x_1)$$

**Ex:** Find the equation of the tangent line to $y = x^5 - 4x^3$ at the point $(-1, 3)$.

$$(x_1, y_1) = (-1, 3)$$

$$y' = 5x^4 - 12x^2$$

$$y'|_{x=-1} = -7$$

$$m = -7$$

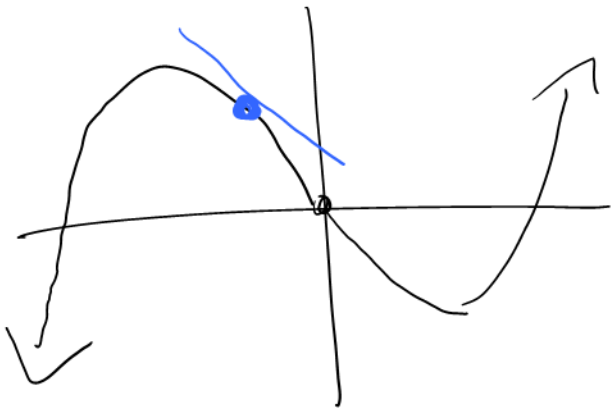
$$y - y_1 = m(x - x_1)$$

$$y - 3 = -7(x + 1)$$

$$y - 3 = -7x - 7$$

$$7x + y + 4 = 0$$

Standard Form $ax + by + C = 0$



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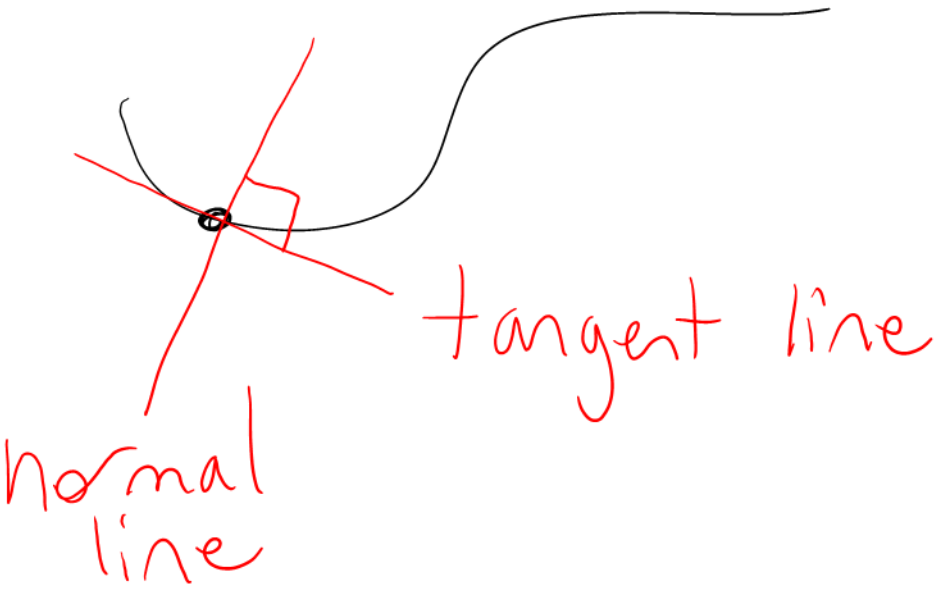



Perpendicular lines have slopes that are negative reciprocals.

e.g. $m_{tan} = 7$ $m_{normal} = \frac{-1}{7}$

$m_{tan} = \frac{2}{3}$ $m_{normal} = \frac{-3}{2}$

$m_{normal} = -\frac{8}{9}$ $m_{tan} = \frac{9}{8}$

$$m_{normal} = \frac{-1}{m_{tan}}$$

$$m_{tan} = \frac{-1}{m_{normal}}$$

Ex: Find the equation of the normal line to $y = (x^2+1)(4x^3+2)$ at $x=-1$.

$y' = (x^2+1)(12x^2) + (4x^3+2)(2x)$

$$y'\Big|_{x=-1} = (2)(12) + (-2)(-2)$$

$$= 28$$

$$m_{tan} = 28$$

$$\boxed{m_{normal} = \frac{-1}{28}}$$

$$\boxed{x_1 = -1}$$

$$x = -1 \rightarrow y = (x^2+1)(4x^3+2)$$

$$= (2)(-2)$$

$$= -4$$

$$\boxed{y_1 = -4}$$

$$y - y_1 = m(x - x_1)$$

$$y + 4 = \frac{-1}{28}(x+1)$$

$$28y + 112 = -x - 1$$

$$x + 28y + 113 = 0$$

**Ex:** Given $y = (2x+1)^2$

Find the equation of:

a) the tangent line with slope 20

$$m_{tan} = 20$$

Find $x_1$ and $y_1$.

$$y' = 2(2x+1)(2)$$

$$m_{tan} = 4(2x+1)$$

Set $m_{tan} = 20$:

$$20 = 4(2x+1)$$

$$5 = 2x+1$$

$$4 = 2x$$

$$2 = x$$

$$\boxed{x_1 = 2}$$

$$x = 2 \rightarrow y = (2x+1)^2$$

$$y = 25$$

$$\boxed{y_1 = 25}$$

$$y - y_1 = m(x - x_1)$$

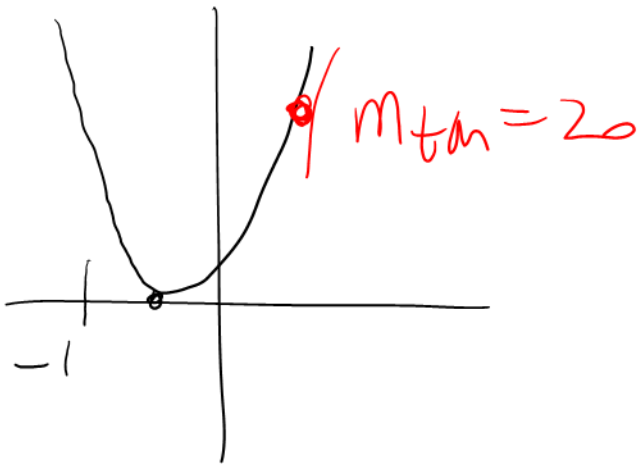
$$y - 25 = 20(x - 2)$$

$$y - 25 = 20x - 40$$

$$-20x + y + 15 = 0$$

OR

$$20x - y - 15 = 0$$



b) the normal line with slope $\frac{1}{4}$

$$m_{tan} = 4(2x+1)$$

$$\boxed{m_{normal} = \frac{1}{4}}$$

Find $x_1$ and $y_1$

$$m_{normal} = \frac{-1}{4(2x+1)}$$

$$\frac{-1}{4(2x+1)} = \frac{1}{4}$$

$$-4 = 4(2x+1)$$

$$-4 = 8x + 4$$

$$-8 = 8x$$

$$x = -1$$

$x = -1 \rightarrow$ $y = (2x+1)^2$

$y = 1$

$$\boxed{x_1 = -1 \qquad y_1 = 1}$$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{1}{4}(x + 1)$$

$$4y - 4 = x + 1$$

$$-x + 4y - 5 = 0$$