23.8 Implicit Differentiation Cont'd

$$\frac{d}{dx}\left[9x^3\right] = 27x^2$$

$$\frac{d}{dx}\left[9\left(4x^2+1\right)^3\right] = 27\left(4x^2+1\right)^2(8x)$$

Suppose y depends on x (y is <u>not</u> constant)

$$\frac{d}{dx}\left[9y^3\right] = 27y^2\frac{dy}{dx}$$

<u>Warm Up</u>: Find $\frac{dy}{dx}$

$$y^2+4 = x^2-y+3x$$

1) Take $\frac{d}{dx}$

$$2y\frac{dy}{dx}+0 = 2x-\frac{dy}{dx}+3$$

2) Solve for $\frac{dy}{dx}$

$$2y\frac{dy}{dx}+\frac{dy}{dx} = 2x+3$$

$$\left[2y+1\right]\frac{dy}{dx} = 2x+3$$

$$\frac{dy}{dx} = \frac{2x+3}{2y+1}$$

<u>Ex</u>: Find $\frac{dy}{dx}$ given:

$$4x^3y+\left(y^3+x\right)^4 = 16$$

$$(4x^3)\,y$$

1) Take $\frac{d}{dx}$

$$(4x^3)\frac{dy}{dx} + y(12x^2) + \boxed{4(y^3+x)^3}(3y^2\frac{dy}{dx}+1) = 0$$

2) Solve for $\frac{dy}{dx}$

$$4x^3\frac{dy}{dx} + 12x^2y + 12(y^3+x)^3y^2\frac{dy}{dx} + 4(y^3+x)^3 = 0$$

$$4x^3\frac{dy}{dx} + 12(y^3+x)^3y^2\frac{dy}{dx} = -12x^2y - 4(y^3+x)^3$$

$$\left[4x^3 + 12(y^3+x)^3y^2\right]\frac{dy}{dx} = -12x^2y - 4(y^3+x)^3$$

$$\frac{dy}{dx} = \frac{-12x^2y - 4(y^3+x)^3}{4x^3 + 12y^2(y^3+x)^3}$$

Ex: Find $\frac{dy}{dx}$ given $x + \sqrt{x}\,y = \sqrt{y}$

Simplify.

$$x + x^{1/2}y = y^{1/2}$$

1) Take $\frac{d}{dx}$

$$1 + x^{1/2}\frac{dy}{dx} + y\left(\tfrac{1}{2}x^{-1/2}\right) = \tfrac{1}{2}y^{-1/2}\frac{dy}{dx}$$

2) Solve for $\frac{dy}{dx}$

$$x^{1/2}\frac{dy}{dx} - \tfrac{1}{2}y^{-1/2}\frac{dy}{dx} = -1 - \tfrac{1}{2}x^{-1/2}y$$

$$\left[x^{1/2} - \tfrac{1}{2}y^{-\frac{1}{2}}\right]\frac{dy}{dx} = -1 - \tfrac{1}{2}x^{-\frac{1}{2}}y$$

$$\frac{dy}{dx} = \frac{\left(-1 - \frac{1}{2}x^{-\frac{1}{2}}y\right)}{\left(x^{1/2} - \frac{1}{2}y^{-\frac{1}{2}}\right)} \cdot \frac{2x^{1/2}y^{1/2}}{2x^{1/2}y^{1/2}}$$

$\boxed{\text{Simplify: Clear fractions and negative exponents}}$

$$\frac{dy}{dx} = \frac{-2x^{1/2}y^{1/2} - y^{3/2}}{2xy^{1/2} - x^{1/2}}$$

$$\boxed{x^a \cdot x^b = x^{a+b}}$$

$$\boxed{\left(\frac{u}{v}\right)' = \frac{vu' - uv'}{v^2}}$$

<u>Ex</u>: Find $\frac{dy}{dx}$ given $x = \frac{x-y}{x+y}$

1) Take $\frac{d}{dx}$

$$1 = \frac{(x+y)\left(1-\frac{dy}{dx}\right) - (x-y)\left(1+\frac{dy}{dx}\right)}{(x+y)^2}$$

2) Solve for $\frac{dy}{dx}$

$$(x+y)^2 = (x+y)\left(1-\frac{dy}{dx}\right) \underbrace{-(x-y)}_{+(-x+y)}\left(1+\frac{dy}{dx}\right)$$

$$(x+y)^2 = \cancel{x} - x\frac{dy}{dx} + y - \cancel{y\frac{dy}{dx}} \cancel{-x} - x\frac{dy}{dx} + y + \cancel{y\frac{dy}{dx}}$$

$$x\frac{dy}{dx} + x\frac{dy}{dx} = 2y - (x+y)^2$$

$$2x\frac{dy}{dx} = 2y - (x+y)^2$$

$$\frac{dy}{dx} = \frac{2y - (x+y)^2}{2x}$$