23.8 Implicit Differentiation

y is an <u>explicit</u> function of x means that we have solved for y

e.g. $y = \frac{x-1}{x+1}$

Otherwise y is an <u>implicit</u> function of x.

e.g. $x^2 + y^2 = 1$

$5x - 3x^2 y = \frac{4}{y}$

Can be impossible to solve for y.

<u>Ex</u>: y is an implicit function of x (y depends on x).

Find:

a) $\frac{d}{dx}[4x^2] = 8x$

b) $\frac{d}{dx}[4y^2] = 8y\frac{dy}{dx}$ Chain Rule

Multiply by $\frac{dy}{dx}$ when differentiating a y-term.

c) $\frac{d}{dx}[x^3 - 2y^2 + y]$

$= 3x^2 - 4y\frac{dy}{dx} + \frac{dy}{dx}$

d) $\frac{d}{dx}[9xy] = \frac{d}{dx}[(9x)y]$

$= 9x\frac{dy}{dx} + y(9)$

$= 9x\frac{dy}{dx} + 9y$

$$\frac{d}{dx}\left[\,4\left(3x^8+1\right)^2\,\right]=8\left(3x^8+1\right)\left(24x^7\right)$$

$$\frac{d}{dx}\left[\,4y^2\,\right]=8y\,\frac{dy}{dx}$$

Ex: Find $\frac{dy}{dx}$ @ $(x,y)=(0,2)$

$$5x-3x^2y+y^2=4$$

1) Take $\frac{d}{dx}$

$$5+\frac{d}{dx}\left[(-3x^2)y\right]+2y\frac{dy}{dx}=0$$

$$5+(-3x^2)\frac{dy}{dx}+y(-6x)+2y\frac{dy}{dx}=0$$

2) Solve for $\frac{dy}{dx}$

$$-3x^2\frac{dy}{dx}+2y\frac{dy}{dx}=-5+6xy$$

$$\left[-3x^2+2y\right]\frac{dy}{dx}=-5+6xy$$

$$\frac{dy}{dx} = \frac{-5 + 6xy}{-3x^2 + 2y}$$

$$\left.\frac{dy}{dx}\right|_{(x,y)=(0,2)} = -\frac{5}{4}$$

Implicit Function