Product Rule $(uv)' = uv' + vu'$

Quotient Rule $\left(\frac{u}{v}\right)' = \frac{vu' - uv'}{v^2}$

Chain Rule $y = 3(4x^7 - 8x + 2)^{11}$

$y' = 33(4x^7 - 8x + 2)^{10}(28x^6 - 8)$

$y = (9x+1)^8$

$y' = 8(9x+1)^7(9)$

23.7 Cont'd

Ex: Find $f'(1)$ for $f(x) = [(3x+2)(9x^2+1)]^4$

(Chain Rule) $f'(x) = 4[(3x+2)(9x^2+1)]^3 \frac{d}{dx}[(3x+2)(9x^2+1)]$

(Product) $= 4[(3x+2)(9x^2+1)]^3 [(3x+2)(18x) + (9x^2+1)(3)]$

$f'(1) = 4[(5)(10)]^3[(5)(18) + (10)(3)]$

$$= 4(50)^3(120)$$

$$= 6 \times 10^7$$

Ex: $y = \dfrac{x}{\sqrt{1-3x}}$. Find y' and simplify.

(Quotient) $$y' = \frac{\sqrt{1-3x}\,(1) - x\,\frac{d}{dx}(1-3x)^{1/2}}{(1-3x)}$$

$$= \frac{\sqrt{1-3x} - x\left[\frac{1}{2}(1-3x)^{-1/2}(-3)\right]}{(1-3x)}$$

Simplify: Clear all fractions and all negative exponents

$$= \left[\frac{\sqrt{1-3x} + \frac{3x}{2}(1-3x)^{-1/2}}{(1-3x)}\right] \frac{2(1-3x)^{1/2}}{2(1-3x)^{1/2}}$$

$$= \frac{2(1-3x) + 3x}{2(1-3x)^{3/2}}$$

$$= \frac{2 - 6x + 3x}{2(1-3x)^{3/2}}$$

$$= \frac{2-3x}{2(1-3x)^{3/2}}$$

Ex: $f(x) = \sqrt{3x^2+6}\,(4-2x)$.

Simplify $f'(x)$.

(Product) $f'(x) = \sqrt{3x^2+6}\,(-2) + (4-2x)\frac{d}{dx}(3x^2+6)^{1/2}$

(Chain) $= -2\sqrt{3x^2+6} + (4-2x)\left[\frac{1}{2}(3x^2+6)^{-\frac{1}{2}}(6x)\right]$

$= -2\sqrt{3x^2+6} + \frac{3x(4-2x)}{\sqrt{3x^2+6}}$

(Common Denominator) $= \frac{-2\sqrt{3x^2+6}\sqrt{3x^2+6}}{\sqrt{3x^2+6}} + \frac{3x(4-2x)}{\sqrt{3x^2+6}}$ $\quad (3x^2+6)$

$= \frac{-6x^2-12+12x-6x^2}{\sqrt{3x^2+6}}$

$= \frac{-12x^2+12x-12}{\sqrt{3x^2+6}}$

Ex: Velocity of a wave in deep water

$$v = k\sqrt{\frac{\ell}{a} + \frac{a}{\ell}}$$

k, a : positive constants

ℓ = wavelength

For what value of ℓ is $\frac{dv}{d\ell} = 0$?

$$v = k\left(\frac{\ell}{a} + a\ell^{-1}\right)^{1/2}$$

$$\frac{dv}{d\ell} = \frac{k}{2}\left(\frac{\ell}{a} + a\ell^{-1}\right)^{-1/2}\left(\frac{1}{a} - a\ell^{-2}\right)$$

Set $\frac{dv}{d\ell} = 0$:

$$\frac{\frac{k}{2}\left(\frac{1}{a} - a\ell^{-2}\right)}{\left(\frac{\ell}{a} + a\ell^{-1}\right)^{1/2}} = 0$$

$$\frac{k}{2}\left(\frac{1}{a} - a\ell^{-2}\right) = 0$$

$$\frac{1}{a} - a\ell^{-2} = 0$$

$$\frac{1}{a} = \cancel{a\ell^{-2}} \quad \frac{a}{\ell^2}$$

$$\ell^2 = a^2$$

$$\ell = \pm a$$

Wavelength is > 0

$$\boxed{\ell = a}$$