

23.6 Cont'd

Product Rule $[uv]' = uv' + vu'$

Quotient Rule $\left[\frac{u}{v}\right]' = \frac{vu' - uv'}{v^2}$

Ex: Find $f'(-3)$

$$f(x) = (2x^2 - x + 1)(4 - 2x - x^2)$$

$$f'(x) = (2x^2 - x + 1)(-2 - 2x) + (4 - 2x - x^2)(4x - 1)$$

$$\begin{aligned} f'(-3) &= (22)(4) + (1)(-13) \\ &= 75 \end{aligned}$$

Ex: Find $f'(1)$

$$f(x) = \frac{2x^2 - x - 1}{x^3 + 2x^2}$$

$$\left[\frac{u}{v}\right]' = \frac{vu' - uv'}{v^2}$$

$$f'(x) = \frac{(x^3 + 2x^2)(4x - 1) - (2x^2 - x - 1)(3x^2 + 4x)}{(x^3 + 2x^2)^2}$$

$$f'(1) = \frac{(3)(3) - (0)(7)}{(3)^2}$$

$$= 1$$

Ex: Find $f'(x)$ and simplify fully.

$$f(x) = \frac{x^2 + 4}{x - 7}$$

$$f'(x) = \frac{(x-7)(2x) - 1(x^2+4)(1)}{(x-7)^2}$$

To simplify: Keep denominator factored.
Expand numerator.

$$= \frac{2x^2 - 14x - x^2 - 4}{(x-7)^2}$$

$$= \frac{x^2 - 14x - 4}{(x-7)^2}$$

Ex: Find $f'(x)$ for

$$f(x) = \frac{(3-7x)(5x+8)}{(4x+1)}$$

Quotient Rule \approx

$$f'(x) = \frac{v u' - u v'}{v^2}$$

$$f'(x) = \frac{(4x+1) [(3-7x)(5x+8)]' - (3-7x)(5x+8)(4)}{(4x+1)^2}$$

$$= \frac{(4x+1) [(3-7x)(5) + (5x+8)(-7)] - (3-7x)(5x+8)(4)}{(4x+1)^2}$$

$$f'(2) = \frac{(9) [-11(5) + 18(-7)] - (-11)(18)(4)}{9^2}$$

$$= \frac{9[-181] + 792}{81}$$

$$= \frac{-837}{81}$$

$$\approx -10.33$$

23.7 General Power Rule and Chain Rule

Before $\frac{d}{dx} [x^n] = nx^{n-1}$ for $n=1, 2, 3, \dots$

General Power Rule

$\frac{d}{dx} [x^n] = nx^{n-1}$ for $n = \text{any rational \#}$

e.g. $n = \frac{2}{3}, \frac{-4}{5}, 0, 2, -3, \dots$

Recap: Exponents

$$\sqrt{x} = x^{1/2}$$

$$\sqrt[3]{x} = x^{1/3}$$

$$\sqrt[10]{x} = x^{1/10}$$

$$\sqrt[7]{x^8} = x^{8/7}$$

$$\frac{1}{x^4} = x^{-4}$$

$$\frac{1}{x} = x^{-1}$$

$$\frac{1}{\sqrt[7]{x^{11}}} = x^{-11/7}$$

Ex: Find $f'(x)$

a) $f(x) = \sqrt{x}$

$$f(x) = x^{1/2}$$

$$f'(x) = \frac{1}{2} x^{-1/2} \quad \text{or} \quad \frac{1}{2\sqrt{x}}$$

b) $f(x) = \frac{1}{x}$

$$f(x) = x^{-1}$$

$$f'(x) = -x^{-2} \quad \text{or} \quad -\frac{1}{x^2}$$

c) $f(x) = 5 \cdot \sqrt[4]{x^3}$

$$f(x) = 5x^{3/4}$$

$$f'(x) = \frac{15}{4} x^{-1/4}$$

d) $f(x) = \frac{12}{\sqrt{x^3}}$

$$f(x) = 12x^{-3/2}$$

$$f'(x) = 12 \left(-\frac{3}{2} x^{-5/2} \right)$$

$$= -18x^{-5/2}$$