23.3 Cont'd

Def

Differentiate $f(x)$: Find $f'(x)$

Def

$f(x)$ is differentiable : $f'(x)$ exists

Last example:

$$f(x) = 2x^3 - \pi$$

$$\vdots$$

$$f'(x) = 6x^2$$

$f'(x) = 6x^2$ exists for all x

$f(x) = 2x^3 - \pi$ is differentiable for all x

Recap: Common Denominators

$$\frac{2}{x+h-1} - \frac{2}{x-1}$$

$$= \frac{2(x-1) - 2(x+h-1)}{(x+h-1)(x-1)}$$

$$= \frac{2x-2-2x-2h+2}{(x+h-1)(x-1)}$$

expand numerator
keep denominator factored

$$= \frac{-2h}{(x+h-1)(x-1)}$$

Ex: $f(x) = x^2 + \frac{4}{x}$ Find $f'(x)$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}\left[(x+h)^2 + \frac{4}{x+h} - x^2 - \frac{4}{x}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[(x+h)^2 - x^2 + \frac{4}{x+h} - \frac{4}{x}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\cancel{x^2} + 2xh + h^2 - \cancel{x^2} + \frac{4x-4(x+h)}{(x+h)x}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[2xh + h^2 + \frac{\cancel{4x}-\cancel{4x}-4h}{(x+h)x}\right]$$

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}}\left[2x + h + \frac{-4}{(x+h)x}\right]$$

$$= 2x - \frac{4}{x^2}$$

Follow-Up:

$f'(x)$ is defined for $x \neq 0$

$f(x)$ is differentiable for $x \neq 0$

Ex: $f(x) = \sqrt{x}$

Find $f'(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(\sqrt{x+h} - \sqrt{x})}{h} \cdot \frac{(\sqrt{x+h} + \sqrt{x})}{(\sqrt{x+h} + \sqrt{x})}$$

Multiply top and bottom by the conjugate radical.

$$= \lim_{h \to 0} \frac{x+h + \cancel{\sqrt{x+h}\sqrt{x}} - \cancel{\sqrt{x}\sqrt{x+h}} - x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{\cancel{h}\, 1}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$= \frac{1}{\sqrt{x} + \sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}$$

$f'(x)$ is defined for $x > 0$

$f(x)$ is differentiable for $x > 0$

Ex: $f(x) = \sqrt{x+3}$

Find $f'(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(\sqrt{x+h+3} - \sqrt{x+3})}{h} \cdot \frac{(\sqrt{x+h+3} + \sqrt{x+3})}{(\sqrt{x+h+3} + \sqrt{x+3})}$$

$$= \lim_{h \to 0} \frac{x+h+3 + \cancel{\sqrt{x+h+3}\sqrt{x+3}} - \cancel{\sqrt{x+3}\sqrt{x+h+3}} - (x+3)}{h(\sqrt{x+h+3} + \sqrt{x+3})}$$

$$= \lim_{h \to 0} \frac{\cancel{h}\ 1}{\cancel{h}(\sqrt{x+h+3} + \sqrt{x+3})}$$

$$= \frac{1}{\sqrt{x+3} + \sqrt{x+3}}$$

$$= \frac{1}{2\sqrt{x+3}}$$

$f'(x)$ exists when $x > -3$

$f(x)$ is differentiable when $x > -3$