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$$\frac{dV}{dt} = -2 \ \frac{m^3}{h}$$

$$\frac{dh}{dt} = ? \quad \text{when } h = 6m$$

$$V = \frac{1}{3}\pi r^2 h$$

$$\boxed{\begin{array}{l} \dfrac{r}{h} = \dfrac{5}{14} \quad \text{by similar triangles} \\ r = \dfrac{5}{14} h \end{array}}$$

$$V = \frac{1}{3}\pi \left(\frac{5}{14} h\right)^2 h$$

$$V = \frac{\pi}{3} \frac{5^2}{14^2} h^3$$

Take $\frac{d}{dt}$: $\quad \dfrac{dV}{dt} = \dfrac{dV}{dh} \dfrac{dh}{dt}$

$$\frac{dV}{dt} = \frac{\pi}{\cancel{3}} \frac{5^2}{14^2} \cancel{3} h^2 \frac{dh}{dt}$$

$\dfrac{dV}{dt} = -2$

$h = 6$: $\quad -2 = \pi \dfrac{5^2}{14^2} (36) \dfrac{dh}{dt}$

$$\frac{-2(14^2)}{\pi\, 5^2 (36)} = \frac{dh}{dt}$$

$$\frac{dh}{dt} \approx -0.14 \ \frac{m}{h}$$

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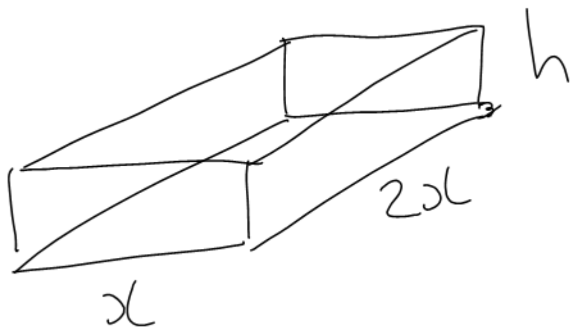

$$x + 2x + h = 140$$

Maximize $V = 2x(x)(h)$

$$= 2x^2 h$$

$$\boxed{\begin{aligned} x + 2x + h &= 140 \\ h &= 140 - 3x \end{aligned}}$$

$$V = 2x^2(140 - 3x)$$

$$V = 280x^2 - 6x^3$$

$$V' = 560x - 18x^2$$

Set $V' = 0$: $\quad 0 = 560x - 18x^2$

$$0 = x(560 - 18x)$$

$x = 0$

MINIMUM VOLUME

$560 - 18x = 0$

$560 = 18x$

$\frac{560}{18} = x$

MAXIMUM VOLUME

$$x = \frac{560}{18} \rightarrow V = 280\left(\frac{560}{18}\right)^2 - 6\left(\frac{560}{18}\right)^3$$

$$\approx 90337 \quad cm^3$$

15 c) $$\int_0^1 \frac{3x^4}{(1+7x^5)^2}\,dx$$

$$\boxed{\begin{aligned} u &= 1+7x^5 \\ \frac{du}{dx} &= 35x^4 \\ du &= 35x^4\,dx \\ \frac{1}{35}\,du &= x^4\,dx \\ x = 0 &\Rightarrow u = 1 \\ x = 1 &\Rightarrow u = 8 \end{aligned}}$$

$$= \frac{3}{35}\int_1^8 u^{-2}\,du$$

$$= \frac{3}{35}\left[-u^{-1}\right]_1^8$$

$$= \frac{3}{35}\left[-\frac{1}{8} + 1\right]$$

$$= 0.075$$

Alternatively:

$$\int_0^1 \frac{3x^4}{(1+7x^5)^2}\,dx$$

$$= \frac{3}{35}\int_{x=0}^{x=1} u^{-2}\,du$$

$$= \frac{3}{35}\left[-u^{-1}\right]_{x=0}^{x=1}$$

$$= \frac{3}{35}\left[\frac{-1}{1+7x^5}\right]_0^1$$

$$= \frac{3}{35}\left[\frac{-1}{8} + 1\right]$$

$$= 0.075$$

$$\int x^7 dx = \frac{x^8}{8} + C$$

$$\int x^4 dx = \frac{x^5}{5} + C$$

$$\int x^{-2} dx = -x^{-1} + C$$

$$\frac{d}{dx}\left[x^7\right] = 7x^6$$