Exam Review

Problems are on the website.

(2) $$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\left(\sqrt{2(x+h)+1} - \sqrt{2x+1}\right)\left(\sqrt{2x+2h+1} + \sqrt{2x+1}\right)}{h\left(\sqrt{2x+2h+1} + \sqrt{2x+1}\right)}$$

$$= \lim_{h\to 0} \frac{2x+2h+1 + \sqrt{\quad}\sqrt{\quad} - \sqrt{\quad}\sqrt{\quad} - (2x+1)}{h\left(\sqrt{2x+2h+1} + \sqrt{2x+1}\right)}$$

$$= \lim_{h\to 0} \frac{2h}{h\left(\sqrt{2x+2h+1} + \sqrt{2x+1}\right)}$$

$$= \frac{2}{\sqrt{2x+1} + \sqrt{2x+1}}$$

$$= \frac{2}{2\sqrt{2x+1}}$$

$$= \frac{1}{\sqrt{2x+1}}$$

(3) $$y = (2x+1)^{2/3}(x^3 - 3x^2)$$

$$[uv]' = uv' + vu'$$

$$y' = (2x+1)^{2/3}(3x^2 - 6x) + (x^3 - 3x^2)\left[\frac{2}{3}(2x+1)^{-1/3}(2)\right]$$

$$y'|_{x=2} = 5^{2/3}(0) - 4\tfrac{2}{3}(5^{-1/3})(2)$$

$$= -\frac{16}{3(5^{1/3})}$$

(4)

$$\left[\frac{u}{v}\right]' = \frac{vu' - uv'}{v^2}$$

$$y = \frac{8x^2+3}{5x+1}$$

$$\frac{dy}{dx} = \frac{(5x+1)(16x) - 5(8x^2+3)(5)}{(5x+1)^2}$$

$$= \frac{80x^2+16x-40x^2-15}{(5x+1)^2}$$

Expand Numerator

$$= \frac{40x^2+16x-15}{(5x+1)^2}$$

(5) Implicit Differentiation

$$\frac{d}{dx}[x^3] = 3x^2$$

$$\frac{d}{dx}[y^3] = 3y^2\frac{dy}{dx}$$ (Chain Rule)

$$\cos(xy) - \sin(3y) = 1 + x^3$$

Take $\frac{d}{dx}$: $-\sin(xy)\left[x\frac{dy}{dx} + y(1)\right] - \cos(3y)\left[3\frac{dy}{dx}\right] = 3x^2$

Solve for $\frac{dy}{dx}$: $-x\sin(xy)\frac{dy}{dx} - y\sin(xy) - 3\cos(3y)\frac{dy}{dx} = 3x^2$

$$-x\sin(xy)\frac{dy}{dx} - 3\cos(3y)\frac{dy}{dx} = 3x^2 + y\sin(xy)$$

$$\left[-x\sin(xy) - 3\cos(3y)\right]\frac{dy}{dx} = 3x^2 + y\sin(xy)$$

$$\frac{dy}{dx} = \frac{3x^2 + y\sin(xy)}{-x\sin(xy) - 3\cos(3y)}$$

(6) $y = \ln\left[x^3(x^2+4)\right]$

LOG RULES $y = \ln x^3 + \ln(x^2+4)$

$$y = 3\ln x + \ln(x^2+4)$$

$$y' = \frac{3}{x} + \frac{1}{x^2+4}(2x)$$

$$y'\Big|_{x=1} = \frac{17}{5} = 3.4$$

$x = 1 \rightarrow y = \ln\left[x^3(x^2+4)\right]$

$$y = \ln 5$$

$$y - y_1 = m(x - x_1)$$

$m = 3.4$, $x_1 = 1$, $y_1 = \ln 5$ $\rightarrow$

$$y - \ln 5 = 3.4(x-1)$$

$$y = 3.4(x-1) + \ln 5 \quad \checkmark$$

$$y = 3.4x - 3.4 + \ln 5 \quad \checkmark$$

Slope-Intercept Form

(7)

$$e^x = \cos x + 1$$

$$\underbrace{e^x - \cos x - 1}_{f(x)} = 0$$

$$f'(x) = e^x + \sin x$$

x_n	$f(x_n)$	$f'(x_n)$	$x_n - \frac{f(x_n)}{f'(x_n)}$
-3	0.0398	-0.0913	-2.56

Radian Mode
Carry 2 extra d.p.