Quiz Wed Dec 6 Section 16.3

16.4 Cont'd

Ex: Solve by finding A^{-1}

$$\begin{cases} x + 2y + 2z = 17 \\ x + y + z = 9 \\ x \quad\quad - 2z = -9 \end{cases}$$

$\vdots$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & -2 & -4 & -1 & 0 & 1 \end{array}\right]$$

$R_1 - 2R_2$

$R_3 + 2R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 2 & 0 \\ 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & 0 & -2 & 1 & -2 & 1 \end{array}\right]$$

(current row) − # (pivot row)

$$\frac{R_3}{-2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 2 & 0 \\ 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 1 & -\frac{1}{2} \end{array}\right]$$

$$R_2 - 1R_3 \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 2 & 0 \\ 0 & 1 & 0 & \frac{3}{2} & -2 & \frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{2} & 1 & -\frac{1}{2} \end{array}\right]$$

$$\underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{I} \qquad \underbrace{\begin{matrix} -1 & 2 & 0 \\ \frac{3}{2} & -2 & \frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \end{matrix}}_{A^{-1}}$$

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 & 2 & 0 \\ \frac{3}{2} & -2 & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 17 \\ 9 \\ -9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

$$(x, y, z) = (1, 3, 5)$$

16.5 Gauss-Jordan Elimination

A system of equations can have:

1 solution

no solution

infinitely-many solutions

We'll focus on "1 solution" systems.

Omit Sugg HW 16.5 # 7, 13, 15, 25

$$\begin{cases} 3x - 4y + z = 25 \\ 2x + 4y + z = -16 \\ x \qquad\quad + 5z = 11 \end{cases}$$

will be written

$$\left[\begin{array}{ccc|c} 3 & -4 & 1 & 25 \\ 2 & 4 & 1 & -16 \\ 1 & 0 & 5 & 11 \end{array}\right]$$

Gauss-Jordan Elimination:

Use row operations from Section 16.3 to solve the system directly.

Ex: Solve the system above.

$$\left[\begin{array}{ccc|c} 3 & -4 & 1 & 25 \\ 2 & 4 & 1 & -16 \\ 1 & 0 & 5 & 11 \end{array}\right]$$

$R_1 \leftrightarrow R_3$
$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & 11 \\ 2 & 4 & 1 & -16 \\ 3 & -4 & 1 & 25 \end{array}\right]$$

$R_2 - 2R_1$

$R_3 - 3R_1$
$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & 11 \\ 0 & 4 & -9 & -38 \\ 0 & -4 & -14 & -8 \end{array}\right]$$

$$\frac{R_2}{4}\quad \left[\begin{array}{ccc|c} 1 & 0 & 5 & 11 \\ 0 & 1 & -9/4 & -38/4 \\ 0 & -4 & -14 & -8 \end{array}\right]$$

$$R_3+4R_2\quad \left[\begin{array}{ccc|c} 1 & 0 & 5 & 11 \\ 0 & 1 & -9/4 & -38/4 \\ 0 & 0 & -23 & -46 \end{array}\right]$$

$$\frac{R_3}{-23}\quad \left[\begin{array}{ccc|c} 1 & 0 & 5 & 11 \\ 0 & 1 & -9/4 & -38/4 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$\begin{array}{l} R_1-5R_3 \\ R_2+\frac{9}{4}R_3 \end{array}\quad \begin{array}{cccc} x & y & z & \# \\ \multicolumn{4}{c}{\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 2 \end{array}\right]} \end{array}$$

$1x+0y+0z=1 \Rightarrow x=1$

$0x+1y+0z=-5 \Rightarrow y=-5$

$z=2$

$(x,y,z)=(1,-5,2)$