16.3 The Inverse of a Matrix Cont'd

Ex: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Find A^{-1}

$\det A = 1(4) - 2(3)$
$= -2$

$$A^{-1} = \frac{1}{-2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$
$$= -\frac{1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

Identity Matrix

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

or $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Finding A^{-1} for 3x3 matrices

$[A \mid I] \rightsquigarrow [I \mid A^{-1}]$

using 3 operations:

1) Swap 2 rows
2) Multiply / divide a row by a nonzero #
3) (Current row) − # (Pivot Row)

<u>Ex</u>: Given $A = \begin{bmatrix} 2 & 10 & 2 \\ 0 & 4 & 1 \\ 2 & 14 & 2 \end{bmatrix}$

Find A^{-1}.

$$\left[\begin{array}{ccc|ccc} 2 & 10 & 2 & 1 & 0 & 0 \\ 0 & 4 & 1 & 0 & 1 & 0 \\ 2 & 14 & 2 & 0 & 0 & 1 \end{array}\right]$$

Get a 1, "the pivot"

$$\frac{R_1}{2} \quad \left[\begin{array}{ccc|ccc} 1 & 5 & 1 & \frac{1}{2} & 0 & 0 \\ 0 & 4 & 1 & 0 & 1 & 0 \\ 2 & 14 & 2 & 0 & 0 & 1 \end{array}\right]$$

Get 0's in rest of Column 1.

$$R_3 - 2R_1 \quad \left[\begin{array}{ccc|ccc} 1 & 5 & 1 & \frac{1}{2} & 0 & 0 \\ 0 & 4 & 1 & 0 & 1 & 0 \\ 0 & 4 & 0 & -1 & 0 & 1 \end{array}\right]$$

(Current row) − # (pivot row)

Get a 1

$$\frac{R_2}{4} \quad \left[\begin{array}{ccc|ccc} 1 & 5 & 1 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 0 & 4 & 0 & -1 & 0 & 1 \end{array}\right]$$

Get 0's in rest of Column 2.

$$1 - 5\left(\tfrac{1}{4}\right)$$

$$\begin{array}{l} R_1 - 5R_2 \\ \\ R_3 - 4R_2 \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{4} & \frac{1}{2} & -\frac{5}{4} & 0 \\ 0 & 1 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 0 & 0 & -1 & -1 & -1 & 1 \end{array}\right]$$

(current row) − # (pivot row)

Get a 1

$$\frac{R_3}{(-1)} \quad \left[\begin{array}{ccc|ccc} 1 & 0 & -1/4 & 1/2 & -5/4 & 0 \\ 0 & 1 & 1/4 & 0 & 1/4 & 0 \\ 0 & 0 & 1 & 1 & 1 & -1 \end{array}\right]$$

Get 0's in Column 3

$$\frac{1}{2} + \frac{1}{4}$$

$$\begin{array}{l} R_1 + \frac{1}{4}R_3 \\ R_2 - \frac{1}{4}R_3 \end{array} \quad \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{I} \left| \underbrace{\begin{matrix} 3/4 & -1 & -\frac{1}{4} \\ -1/4 & 0 & 1/4 \\ 1 & 1 & -1 \end{matrix}}_{A^{-1}} \right]$$