16.2 Matrix Multiplication Cont'd

Ex: $A = \begin{bmatrix} 4 & -1 \\ 6 & 3 \end{bmatrix}$ $\quad B = \begin{bmatrix} m & -1 \\ 2 & 2m \end{bmatrix}$

Find:

a) $A - 3B$

$$= \begin{bmatrix} 4 & -1 \\ 6 & 3 \end{bmatrix} + \begin{bmatrix} -3m & 3 \\ -6 & -6m \end{bmatrix}$$

$$= \begin{bmatrix} 4-3m & 2 \\ 0 & 3-6m \end{bmatrix}$$

b) AB

$$= \begin{bmatrix} 4 & -1 \\ 6 & 3 \end{bmatrix} \begin{bmatrix} m & -1 \\ 2 & 2m \end{bmatrix}$$

$$= \begin{bmatrix} 4m-2 & -4-2m \\ 6m+6 & -6+6m \end{bmatrix}$$

Identity Matrix

(2×2) $\quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(3×3) $\quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

FACT

$IA = A$ for any matrix A

Ex: Confirm that $IA = A$ for

$$A = \begin{bmatrix} 9 & 8 \\ 2 & 3 \end{bmatrix}$$

$$IA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 9 & 8 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 8 \\ 2 & 3 \end{bmatrix}$$

$$= A \checkmark$$

Def

For a square matrix A (2×2 or 3×3),

A^{-1} has the property that:

$$AA^{-1} = I \quad \text{and} \quad A^{-1}A = I.$$

A^{-1} is called <u>the inverse of A</u>.

<u>Real Numbers</u>
$3^{-1} \cdot 3 = 1$
<u>Matrices</u>
$A^{-1}A = \underline{I}$

<u>Note</u>: Some square matrices don't have inverses.

<u>Ex</u>: $A = \begin{bmatrix} 1 & -4 \\ -2 & 9 \end{bmatrix}$

Check that $\begin{bmatrix} 9 & 4 \\ 2 & 1 \end{bmatrix} = A^{-1}$

Check $A^{-1}A = I$ or $AA^{-1} = I$.

$$A^{-1}A = \begin{bmatrix} 9 & 4 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & -4 \\ -2 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$= I$ ✓

Preview of 16.4

Want to solve a system of equations.

$$AX = B$$

$$\underbrace{A^{-1}A}X = A^{-1}B$$

$$\underbrace{IX} = A^{-1}B$$

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \# \\ \# \end{bmatrix}$$

or

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \# \\ \# \\ \# \end{bmatrix}$$

16.3 The Inverse of a Matrix

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then the <u>determinant of A</u>

is $\det A = ad - bc$

$$A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \qquad \circledast$$

<u>Comments</u>: 1) If $\det A = 0$ then A^{-1} does not exist.

2) Formula only works for 2×2 matrices.

Ex: Find A^{-1}

a) $A = \begin{bmatrix} 9 & 6 \\ 2 & 1 \end{bmatrix}$

$\det A = 9(1) - 6(2)$
$= -3$

$A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$= \frac{1}{-3} \begin{bmatrix} 1 & -6 \\ -2 & 9 \end{bmatrix}$

$= -\frac{1}{3} \begin{bmatrix} 1 & -6 \\ -2 & 9 \end{bmatrix}$

Optional Check: $A^{-1}A = -\frac{1}{3} \begin{bmatrix} 1 & -6 \\ -2 & 9 \end{bmatrix} \begin{bmatrix} 9 & 6 \\ 2 & 1 \end{bmatrix}$

$= -\frac{1}{3} \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= I$ ✓

b) $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

$\det A = 1(4) - 2(2)$
$= 0$

A^{-1} is undefined

c) $A = \begin{bmatrix} -2 & 7 \\ 1 & -3 \end{bmatrix}$

$\det A = (-2)(-3) - 7(1)$
$= -1$

$A^{-1} = \frac{1}{-1} \begin{bmatrix} -3 & -7 \\ -1 & -2 \end{bmatrix}$

$= \begin{bmatrix} 3 & 7 \\ 1 & 2 \end{bmatrix}$

(Could Check $A^{-1}A = I$)

d) $A = \begin{bmatrix} 2c & -3 \\ 4 & c \end{bmatrix}$

$\det A = 2c(c) - (-3)(4)$
$= 2c^2 + 12$

$A^{-1} = \frac{1}{2c^2+12} \begin{bmatrix} c & 3 \\ -4 & 2c \end{bmatrix}$