Review

Ex: Find y'

a) $y = \ln(2x+1) - \ln(x-1)$

$$y' = \frac{1}{2x+1}(2) - \frac{1}{x-1}(1)$$

$$= \frac{2}{2x+1} - \frac{1}{x-1}$$

$$\boxed{\begin{aligned} y &= \log_3 x^2 \\ y' &= \frac{1}{\ln 3}\frac{1}{x^2}(2x) \\ &= \frac{2}{x\ln 3} \end{aligned}}$$

b) $y = \tan^{-1}(6x) - 3\sin^{-1} x^3$

$$y' = \frac{1}{1+(6x)^2}(6) - 3 \cdot \frac{1}{\sqrt{1-(x^3)^2}}(3x^2)$$

$$= \frac{6}{1+36x^2} - \frac{9x^2}{\sqrt{1-x^6}}$$

c) $y = 2^{\sqrt{x}}$

$$y' = (\ln 2)\, 2^{\sqrt{x}} \left(\frac{1}{2}x^{-1/2}\right)$$

$$= \frac{2^{\sqrt{x}} \ln 2}{2\sqrt{x}}$$

d) $y = 6^{3x+4}$

$$y' = (\ln 6)\, 6^{3x+4}(3)$$

$$= (3\ln 6)\, 6^{3x+4}$$

Ex: $$\int_0^2 \frac{x^2}{\sqrt{3x^3+1}^{\,3}}\,dx$$

$$u = 3x^3+1$$
$$\frac{du}{dx} = 9x^2$$
$$du = 9x^2\,dx$$
$$\frac{1}{9}du = x^2\,dx$$
when $x=0,\ u=1$
$x=2,\ u=25$

$$= \frac{1}{9}\int_1^{25} u^{-3/2}\,du$$
$$= \frac{1}{9}\left[-2u^{-1/2}\right]_1^{25}$$
$$= -\frac{2}{9}\left[u^{-1/2}\right]_1^{25}$$
$$= -\frac{2}{9}\left[\frac{1}{5}-1\right]$$
$$= -\frac{2}{9}\left(-\frac{4}{5}\right)$$
$$= \frac{8}{45}$$

Ex: Find $\int\left(-4x^7-2-\frac{2}{\sqrt{x}}\right)dx$

$$= \int\left(-4x^7-2-2x^{-1/2}\right)dx$$
$$= -\frac{4x^8}{8}-2x-2\left(2x^{1/2}\right)+C$$
$$= -\frac{x^8}{2}-2x-4x^{1/2}+C$$

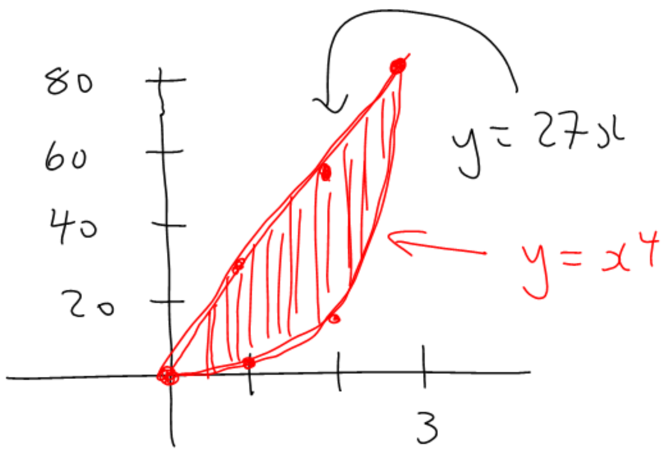
Ex: Find the area between $y = x^4$ and $y = 27x$.

Intersection $y = y$

$$x^4 = 27x$$
$$x^4 - 27x = 0$$
$$x(x^3 - 27) = 0$$

$x = 0$

$$x^3 - 27 = 0$$
$$x^3 = 27$$
$$x = \sqrt[3]{27}$$
$$x = 3$$



x	$y = x^4$
0	0
1	1
2	16
3	81

$$A = \int_a^b (y_t - y_b)\,dx$$
$$= \int_0^3 (27x - x^4)\,dx$$
$$= \left[\frac{27x^2}{2} - \frac{x^5}{5}\right]_0^3$$
$$= \frac{27(9)}{2} - \frac{243}{5} - (0)$$
$$= 72.9$$

Ex: Given $a(t) = -2t$.
Initial velocity $= 10$ m/s.
Stopping time? Stopping distance?

$$v(t) = \int -2t \, dt$$

$$\boxed{v(t) = -t^2 + C_1}$$

$v = 10$
$t = 0$: $10 = 0 + C_1$

$$C_1 = 10$$

$$\boxed{v(t) = -t^2 + 10}$$

$$s(t) = \int (-t^2 + 10) \, dt$$

$$\boxed{s(t) = -\frac{t^3}{3} + 10t + C_2}$$

$s(0) = 0$ unless otherwise specified

$s = 0$
$t = 0$: $0 = 0 + 0 + C_2$

$$C_2 = 0$$

$$\boxed{s(t) = -\frac{t^3}{3} + 10t}$$

Stopping Time: $v = 0$

$$-t^2 + 10 = 0$$

$$10 = t^2$$

$$t^2 = 10$$
$$t = \pm\sqrt{10}$$
$$t = \sqrt{10} \quad s$$

Stopping Distance: $t = \sqrt{10} \longrightarrow s(t) = -\frac{t^3}{3} + 10t$

$$s(\sqrt{10}) = -\frac{\sqrt{10}^3}{3} + 10\sqrt{10}$$
$$\approx 21 \quad m$$

Ex: Estimate $\int_0^1 e^{x^2}\,dx$ using Simpson's Rule with $n=4$.

$$\Delta x = \frac{b-a}{n} = \frac{1-0}{4} = 0.25$$

x	$y = e^{x^2}$
0	1
0.25	$e^{0.0625}$
0.5	$e^{0.25}$
0.75	$e^{0.5625}$
1	e

$$\int_0^1 e^{x^2}\,dx \approx \frac{\Delta x}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\right]$$
$$\approx \frac{0.25}{3}\left[1 + 4e^{0.0625} + 2e^{0.25} + 4e^{0.5625} + e\right]$$

≈ 1.46