

Quiz Wed Nov. 22 Section 26.2

Test Thurs Nov. 23

27.3, 27.5, 27.6, 25.1-6, 26.1-2

(27.8 won't be tested)

Bring calculator  
music earplugs

Practice Problems on website

6 Questions

## 26.6 Applications of Integration (Part 2)

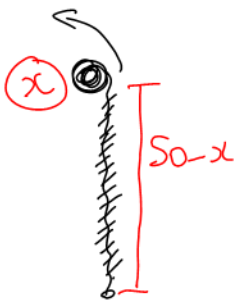
Work  $W = F \cdot d$  when force is constant

More generally:

$$W = \int_a^b F(x) dx$$

where  $F(x) =$  force applied at distance  $x$

Ex: Find the work done in winding up  
30m of a 50m chain that weighs  
2 N/m.



Let  $x =$  length of chain already  
wound up (m)

$$0 \leq x \leq 30$$

Note:  $x$  must be increasing  
(otherwise we get a sign error)

$$F = 2(50 - x) \quad \frac{N}{m} \cdot m$$

$$= 100 - 2x$$

$$W = \int_0^{30} (100 - 2x) dx$$

$$= [100x - x^2]_0^{30}$$

$$= 3000 - 900 - (0)$$

$$= 2100 \quad N \cdot m \text{ or } J$$

Ex: Gravitational attraction between 2 objects is  $F = \frac{k}{x^2}$ , where  $x$  = distance between objects and  $k$  is constant.

Find work done in moving the objects from 20m apart to 40m apart.

$$W = \int_a^b F(x) dx$$

$$= \int_{20}^{40} \frac{k}{x^2} dx$$

$$= -kx^{-1} \Big|_{20}^{40}$$

$$= -k(0.025) + k(0.05)$$

$$= 0.025k \quad N \cdot m \text{ or } J$$

Ex: A spring of natural length 1m requires a force of 8N to stretch it to 3m. Find the work done in stretching it to 3.5m.

2m  
of stretch

2.5m  
of stretch

Hooke's Law  $F = kx$

force (N)      spring constant ( $\frac{N}{m}$ )      amount of stretch (m)

$$8 = k(2)$$

$$k = 4 \frac{N}{m}$$

$$F = kx$$

$$F = 4x$$

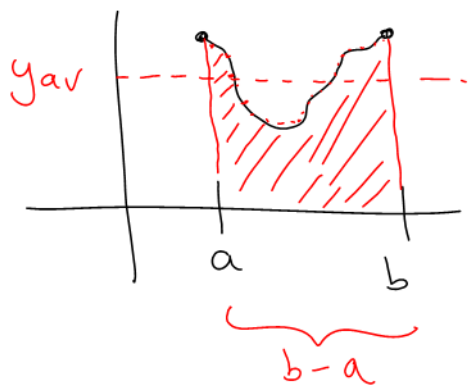
$$0 \leq x \leq 2.5$$

$$W = \int_0^{2.5} 4x \, dx$$
$$= [2x^2]_0^{2.5}$$

$$= 12.5 \text{ J or N}\cdot\text{m}$$

Average Value of a Function  
over the interval  $a \leq x \leq b$

$$y_{\text{av}} = \frac{1}{b-a} \int_a^b y \, dx$$



Ex: Find the average value of  $y = x^4$  from  $x=0$  to  $x=2$ .

$$y_{av} = \frac{1}{b-a} \int_a^b y dx$$

$$= \frac{1}{2} \int_0^2 x^4 dx$$

$$= \frac{1}{2} \left[ \frac{x^5}{5} \right]_0^2$$

$$= \frac{1}{2} \left[ \frac{32}{5} - 0 \right]$$

$$= \frac{32}{10} \text{ or } 3.2$$