26.4 Cont'd

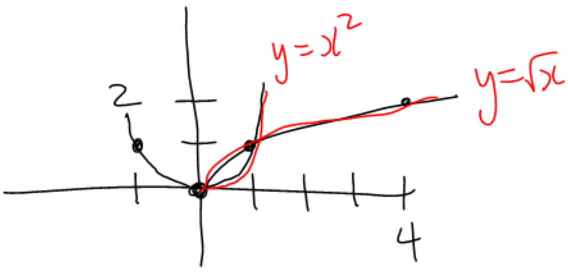
$(\bar{x}, \bar{y})$ = centroid

$$\bar{x} = \frac{1}{A} \int_A x_e \, dA \qquad \bar{y} = \frac{1}{A} \int_A y_e \, dA$$

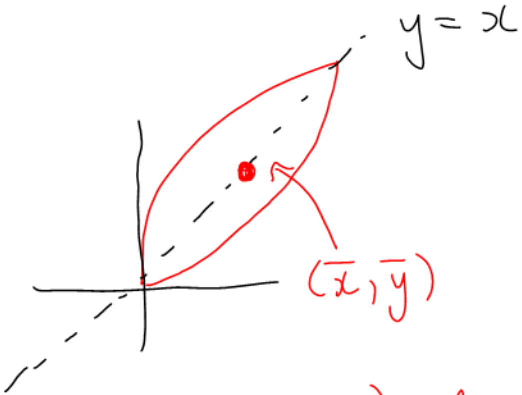
where (x_e, y_e) = Centre of a slice

dA = area of a slice

Ex: Consider the region bounded by $y = \sqrt{x}$ and $y = x^2$. Centroid?



x	$y=\sqrt{x}$
0	0
1	1
4	2



By symmetry, $\bar{y} = \bar{x}$.

Find $\bar{x}$: 1) Area

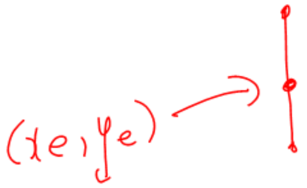
$$A = \int_0^1 \underbrace{(\sqrt{x} - x^2)\,dx}_{dA}$$

$$= \left[\frac{2}{3}x^{3/2} - \frac{x^3}{3}\right]_0^1$$

$$= \left(\frac{2}{3} - \frac{1}{3}\right) - 0$$

$$= \frac{1}{3}$$

2) x_e



$$x_e = x$$

3) $\int_A x_e \, dA$

$$= \int_0^1 x\left(\sqrt{x} - x^2\right) dx$$

$$= \int_0^1 \left(x^{3/2} - x^3\right) dx$$

$$= \left[\frac{2}{5}x^{5/2} - \frac{x^4}{4}\right]_0^1$$

$$= \left(\frac{2}{5} - \frac{1}{4}\right) - 0$$

$$= \frac{3}{20}$$

4) $\bar{x}$

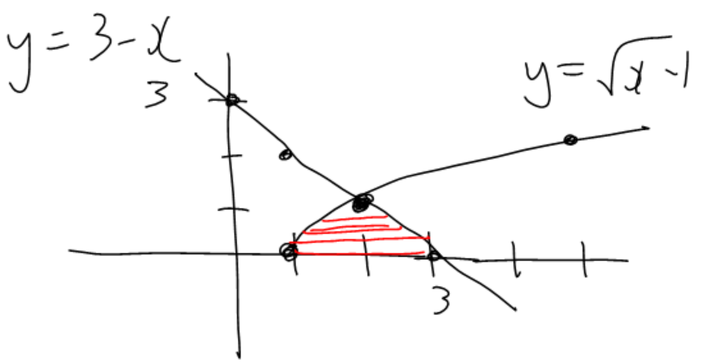
$$\bar{x} = \frac{1}{A} \int_A x_e \, dA$$

$$= 3\left(\frac{3}{20}\right)$$

$$= \frac{9}{20}$$

$$(\bar{x}, \bar{y}) = \left(\frac{9}{20}, \frac{9}{20}\right)$$

Ex: Consider the region bounded by $y = 3 - x$, $y = \sqrt{x-1}$ and $y = 0$. Find $\bar{y}$.



x	$y=\sqrt{x-1}$
1	0
2	1
5	2

1) Area (horizontal slices)

$y = 3 - x$	$y = \sqrt{x-1}$
$x + y = 3$	$y^2 = x - 1$
$x = 3 - y$	$y^2 + 1 = x$
$x_r = 3 - y$	$x_\ell = y^2 + 1$

$$A = \int_0^1 (x_r - x_\ell)\,dy$$

$$= \int_0^1 [3 - y - (y^2+1)]\,dy$$

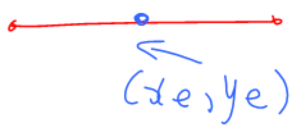
$$= \int_0^1 (3 - y - y^2 - 1)\,dy$$

$$= \int_0^1 \underbrace{\boxed{(2 - y - y^2)\,dy}}_{dA}$$

$$= \left[2y - \frac{y^2}{2} - \frac{y^3}{3}\right]_0^1$$

$$= \left(2 - \frac{1}{2} - \frac{1}{3}\right) - 0$$

$$= \frac{7}{6}$$

2) y_e 

$y_e = y$

3) $\int_A y_e \, dA$

$= \int_0^1 y\,(2-y-y^2)\,dy$

$= \int_0^1 (2y-y^2-y^3)\,dy$

$= \left[y^2 - \frac{y^3}{3} - \frac{y^4}{4} \right]_0^1$

$= \left(1-\frac{1}{3}-\frac{1}{4}\right) - 0$

$= \frac{5}{12}$

4) $\bar{y}$

$\bar{y} = \frac{1}{A}\int_A y_e \, dA$

$= \frac{\left(\frac{5}{12}\right)}{\left(\frac{7}{6}\right)}$

$= \frac{6}{7}\cdot\frac{5}{12}$

$= \frac{5}{14}$