$$\int_1^2 x(4-x^2)^4\,dx$$

$$u = 4-x^2$$

$$\frac{du}{dx} = -2x$$

$$du = -2x\,dx \qquad -\tfrac{1}{2}du = x\,dx$$

$$x=1 \Rightarrow u=3$$

$$x=2 \Rightarrow u=0$$

$$= -\frac{1}{2}\int_3^0 u^4\,du$$

$$= -\frac{1}{2}\left[\frac{u^5}{5}\right]_3^0$$

$$= -\frac{1}{2}\left[0 - \frac{3^5}{5}\right]$$

$$= \frac{3^5}{10}$$

$$= 24.3$$

Test 3

Thurs Nov 23

27.3, 27.5–6, 25.1–6, 26.1–2

(27.8 won't be tested)

$\boxed{\text{6 Questions}}$

Bring: calculator, music/earplugs

Practice Questions on website

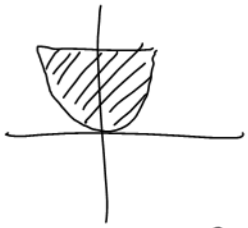
Final Exam

Sat Dec 16
9am - noon
TEC 174

* Look at bus schedule in advance *

26.4 Centroids

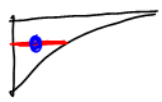
Thin metal plate of constant density and thickness.



<u>Centroid</u>: Centre of mass

Notation: $(\bar{x}, \bar{y})$

We'll use vertical or horizontal slices.
Let (x_e, y_e) = the centre of a slice.

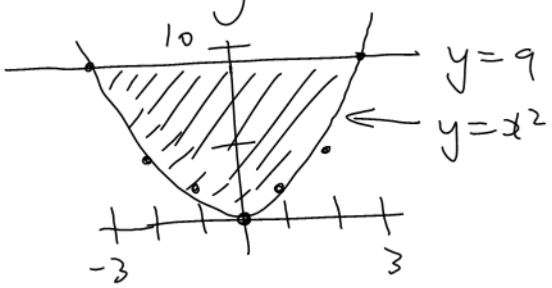
<u>Vertical Slices</u>	<u>Horizontal Slices</u>
 $x_e = x$ $y_e = \dfrac{y_b + y_t}{2}$	 $x_e = \dfrac{x_\ell + x_r}{2}$ $y_e = y$

$$\boxed{\begin{aligned} \bar{x} &= \frac{1}{A}\int_A x_e \, dA \\ \bar{y} &= \frac{1}{A}\int_A y_e \, dA \end{aligned}}$$

area of a slice

Units Analysis: $m = \frac{1}{m^2}(m)(m^2)$ —

Ex: Consider the region bounded by $y=x^2$ and $y=9$. Find the centroid.



$\bar{x} = 0$ by symmetry

$\bar{y}$: 1) Find A

$$A = \int_a^b (y_t - y_b)\,dx$$

$$= \int_{-3}^{3} \boxed{(9-x^2)\,dx} \quad dA$$

$$= \left[9x - \frac{x^3}{3}\right]_{-3}^{3}$$

$$= [27-9] - [-27+9]$$

$$= 36$$

2) y_e for a vertical slice

$$y_e = \frac{y_b + y_t}{2}$$

$$= \frac{x^2+9}{2}$$

3) $\int_A y_e\,dA$

$$= \int_{-3}^{3} \frac{x^2+9}{2} (9 - x^2) \, dx$$

$$= \frac{1}{2} \int_{-3}^{3} (x^2+9)(9-x^2) \, dx$$

$$= \frac{1}{2} \int_{-3}^{3} (\cancel{9x^2} - x^4 + 81 - \cancel{9x^2}) \, dx$$

$$= \frac{1}{2} \left[\frac{-x^5}{5} + 81x \right]_{-3}^{3}$$

$$= \frac{1}{2} \left[\left(\frac{-243}{5} + 243 \right) - \left(\frac{243}{5} - 243 \right) \right]$$

$$= \frac{972}{5}$$

4) $\bar{y} = \frac{1}{A} \int_A y_e \, dA$

$$= \frac{1}{36} \left(\frac{972}{5} \right)$$

$$= \frac{27}{5} \quad \text{or} \quad 5.4$$

$$(\bar{x}, \bar{y}) = (0, 5.4)$$