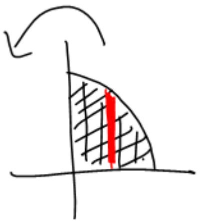
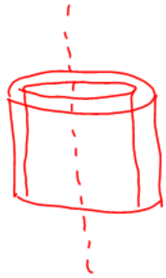
26.3 Volumes Cont'd

Shell Method

Take a slice parallel to the axis of revolution.

Produces a hollow shell (toilet paper roll)



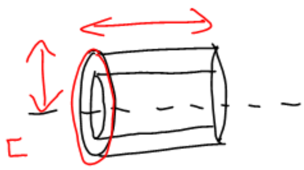


Volume of hollow shell

$$dV = 2\pi \cdot \text{radius} \cdot \text{height} \cdot \text{thickness}$$

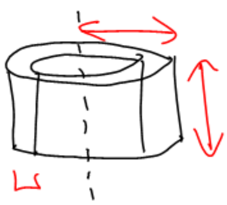
$$V = \int dV$$

About x-axis



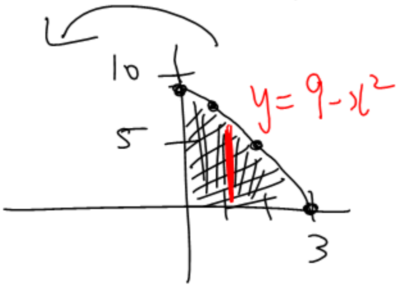
$$dV = 2\pi y x \, dy$$

About y-axis

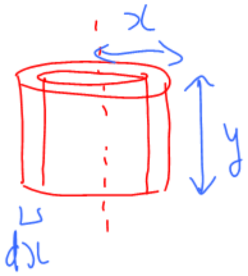


$$dV = 2\pi x y \, dx$$

Ex: Revolve first-quadrant region bounded by $y = 9 - x^2$ about y-axis. Find the volume using shells.



x	$y = 9 - x^2$
0	9
1	8
2	5
3	0



$$dV = 2\pi r h t$$
$$= 2\pi x y \, dx$$

$$V = \int dV$$
$$= \int 2\pi x y \, dx$$
$$= 2\pi \int_0^3 x(9 - x^2)\, dx$$
$$= 2\pi \int_0^3 (9x - x^3)\, dx$$
$$= 2\pi \left[\frac{9x^2}{2} - \frac{x^4}{4} \right]_0^3$$
$$= 2\pi \left[\left(\frac{81}{2} - \frac{81}{4} \right) - 0 \right]$$
$$= \frac{81\pi}{2}$$

ASIDE

a) $\int x^2\,dx = \dfrac{x^3}{3} + C$

b) $\displaystyle\int_1^2 x^2\,dx = \frac{x^3}{3}\Big|_1^2 = \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$

c) $\displaystyle\int_0^1 x\left(100-x^2\right)^9\,dx$

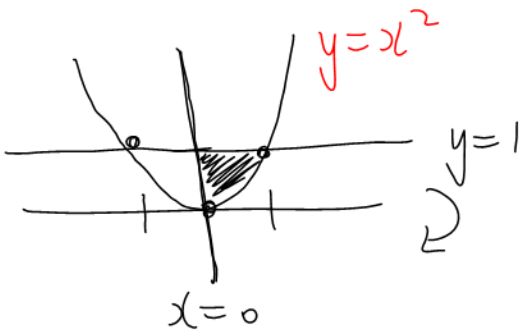
$$\boxed{\begin{aligned} u &= 100-x^2 \\ du &= -2x\,dx \\ -\tfrac{1}{2}du &= x\,dx \\ x=0 &\Rightarrow u=100 \\ x=1 &\Rightarrow u=99 \end{aligned}}$$

$$= -\frac{1}{2}\int_{100}^{99} u^9\,du$$

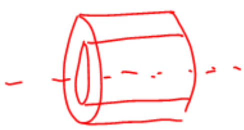
$\vdots$

$=$ positive #

Ex: Revolve first-quadrant region bounded by $y=x^2$, $y=1$, $x=0$ about x-axis.



Cannot use disks (area is not touching axis of revolution).
Must use shells.



$dV = 2\pi y x\,dy$

$$V = \int 2\pi y x \, dy$$

$y = x^2$

$x^2 = y$

$x = \pm\sqrt{y}$

$x = -\sqrt{y}$ $\quad$ $x = \sqrt{y}$

$\boxed{x = \sqrt{y}}$

$$V = 2\pi \int_0^1 y\sqrt{y} \, dy$$

$$= 2\pi \int_0^1 y^{3/2} \, dy$$

$$= 2\pi \left[\frac{2}{5} y^{5/2} \right]_0^1$$

$$= 2\pi \left(\frac{2}{5} \right)$$

$$= \frac{4\pi}{5}$$