Quiz Wed Nov 15 Section 25.4
Wed Nov 22 26.2

Test 3 Thurs Nov 23
27.3, 27.5-6, 25.1-6, 26.1-2
(27.8 won't be tested)

26.3 Volumes by Integration

Revolve an area about an axis
to produce a solid.


solid cylinder

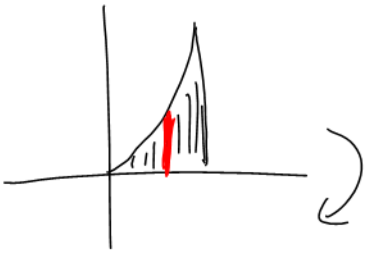

solid cylinder

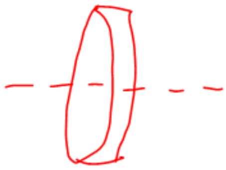

solid cone

Goal: Calculate volumes

Disk Method

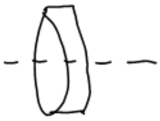
Take a slice perpendicular to the axis
of revolution. Produces a solid disk (hockey puck).

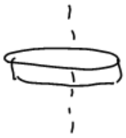




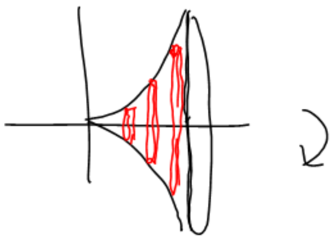
Volume of disk

$$dV = \pi \cdot \text{radius}^2 \cdot \text{thickness}$$

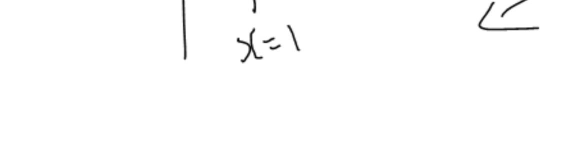
About x-axis  $dV = \pi y^2 \, dx$

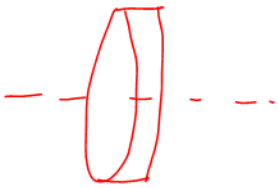
About y-axis  $dV = \pi x^2 \, dy$

$$V = \int dV$$



Ex: Revolve region bounded by $y = x^2$, $y = 0$, $x = 1$ about x-axis. Volume?





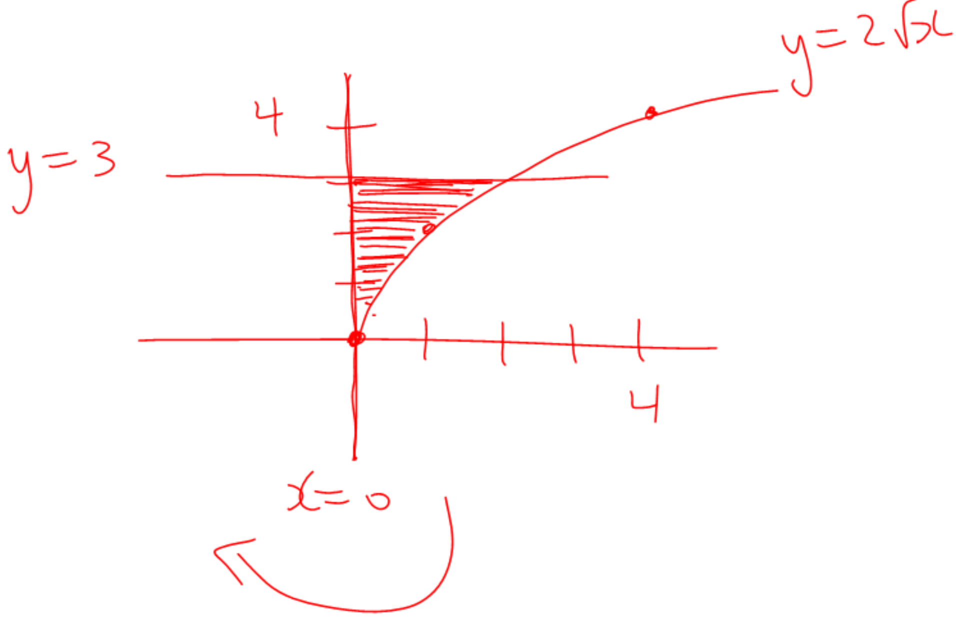
solid disk (hockey puck)

$$dV = \pi \cdot \text{radius}^2 \cdot \text{thickness}$$
$$= \pi y^2 \, dx$$

$$
\begin{aligned}
V &= \int dV \\
&= \int \pi y^2 \, dx \\
&= \pi \int_0^1 (x^2)^2 \, dx \\
&= \pi \int_0^1 x^4 \, dx \\
&= \pi \left[\frac{x^5}{5} \right]_0^1 \\
&= \pi \left[\frac{1}{5} - 0 \right] \\
&= \frac{\pi}{5}
\end{aligned}
$$

<u>Ex</u>: Revolve region bounded by $y = 2\sqrt{x}$, $x = 0$, $y = 3$ about y-axis. Volume?

x	$y = 2\sqrt{x}$
0	0
1	2
4	4



solid disk

$$dV = \pi \cdot r^2 \cdot t$$

$$= \pi x^2 \, dy$$

$$V = \int dV$$

$y = 2\sqrt{x}$

$\frac{y}{2} = \sqrt{x}$

$\frac{y^2}{4} = x$

$x = \frac{y^2}{4}$

$$= \int \pi x^2 \, dy$$

$$= \pi \int_0^3 \left(\frac{y^2}{4}\right)^2 dy$$

$$= \pi \int_0^3 \frac{y^4}{16} \, dy$$

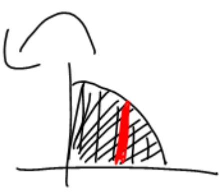
$$= \pi \left[\frac{y^5}{80}\right]_0^3$$

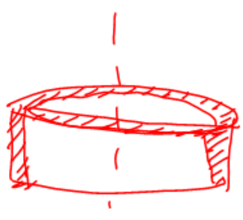
$$= \pi \left[\frac{243}{80} - 0\right]$$

$$= \frac{243\pi}{80}$$

Shell Method

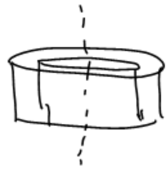
Take a slice parallel to axis of revolution.
Produces a hollow shell

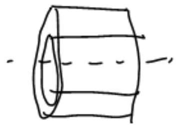




(toilet paper roll)
hollow shell

Volume of hollow shell $dV = 2\pi \cdot \text{radius} \cdot \text{height} \cdot \text{thickness}$

About y-axis  $dV = 2\pi x y \, dx$

About x-axis  $dV = 2\pi y x \, dy$