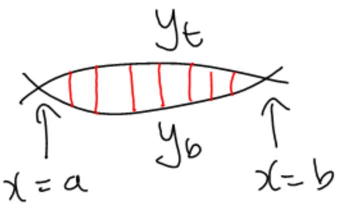
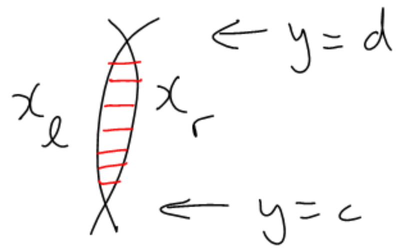
26.2 Cont'd

Vertical Slices



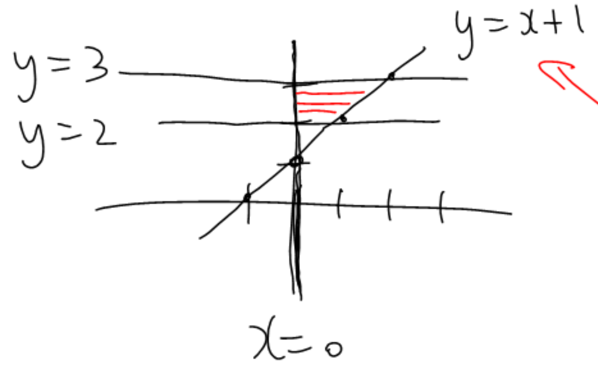
$$A = \int_a^b (y_t - y_b)\,dx$$

Horizontal Slices



$$A = \int_c^d (x_r - x_\ell)\,dy$$

Ex: Area bounded by $y = x+1$, $x = 0$, $y = 2$ and $y = 3$?



$x_\ell = 0$

Horizontal Slices

$$\boxed{\begin{aligned} y &= x+1 \\ y-1 &= x \\ x_r &= y-1 \end{aligned}}$$

$$A = \int_c^d (x_r - x_\ell)\,dy$$

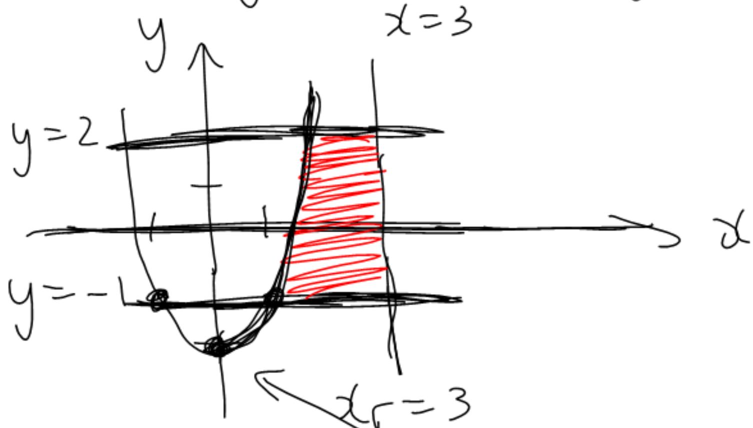
$$= \int_2^3 (y - 1 - 0)\,dy$$

$$= \left[\frac{y^2}{2} - y \right]_2^3$$

$$= \left[\frac{9}{2} - 3 \right] - \left[2 - 2 \right]$$

$$= \frac{3}{2}$$

<u>Ex:</u> Area bounded by $y = x^2 - 2$, $x = 3$, $y = -1$ and $y = 2$?



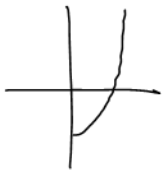
x	$y = x^2 - 2$
-1	-1
0	-2
1	-1

$$y = x^2 - 2$$

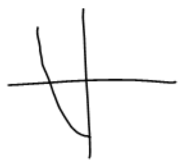
$$y + 2 = x^2$$

$$x^2 = y + 2$$

$$x = \pm\sqrt{y+2}$$



$x = \sqrt{y+2}$



$x = -\sqrt{y+2}$

$$x_l = \sqrt{y+2}$$

$$A = \int_{-1}^{2} \left(3 - \sqrt{y+2}\right) dy$$

$$u = y+2$$
$$du = dy$$
$$y=-1 \Rightarrow u=1$$
$$y=2 \Rightarrow u=4$$

$$= \int_1^4 (3 - u^{1/2})\,du$$

$$= \left[3u - \frac{2}{3}u^{3/2}\right]_1^4$$

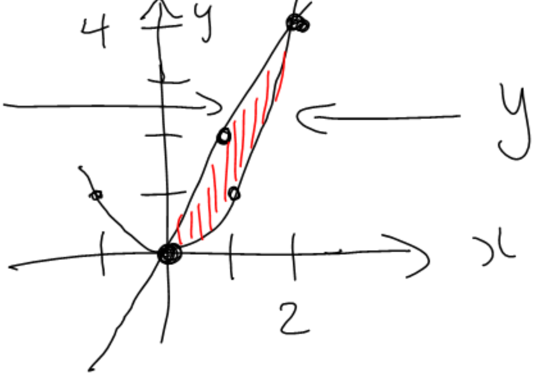
$$= \left[12 - \frac{2}{3}(8)\right] - \left[3 - \frac{2}{3}\right]$$

$$= \frac{13}{3}$$

Ex: Set up area bounded by $y = x^2$ and $y = 2x$ using:

a) vertical slices

$y_t = 2x$



$y_b = x^2$

$$A = \int_a^b (y_t - y_b)\,dx$$

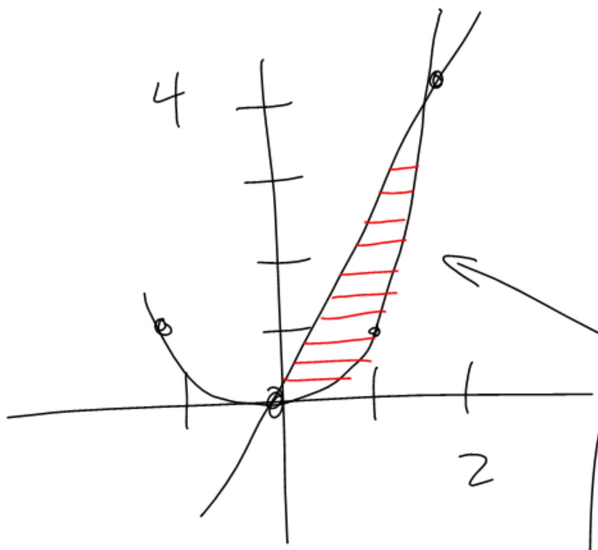
$$A = \int_0^2 (2x - x^2)\,dx$$

b) horizontal slices

$y = 2x$

$\frac{y}{2} = x$

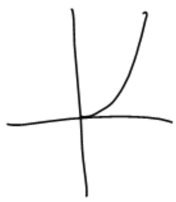
$x_l = \frac{y}{2}$



$y = x^2$

$x^2 = y$

$x = \pm\sqrt{y}$



$x = \sqrt{y}$

$x = -\sqrt{y}$

$x_r = \sqrt{y}$

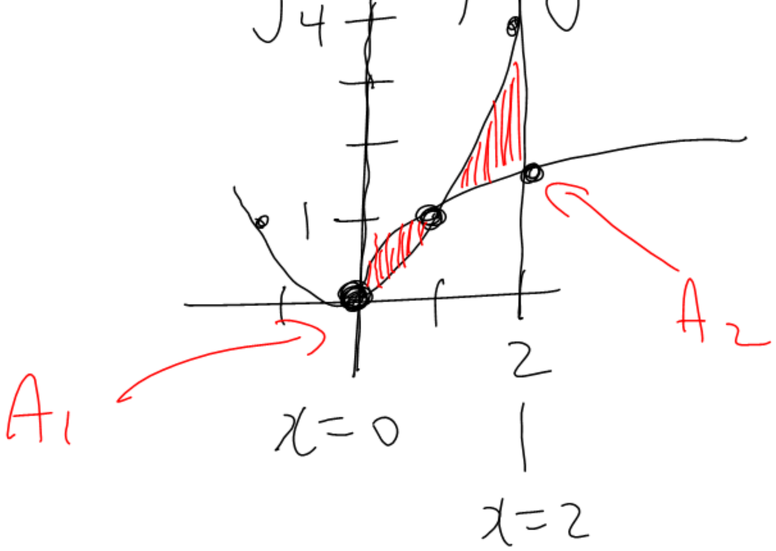
$$A = \int_c^d (x_r - x_\ell)\, dy$$

$$= \int_0^4 \left(\sqrt{y} - \frac{y}{2} \right) dy$$

ASIDE The value is $A = \frac{4}{3}$

Ex: Total area bounded by

$y = x^2$, $y = \sqrt{x}$, $x = 0$ and $x = 2$?



x	$y = \sqrt{x}$
0	0
1	1
2	1.4

$$\boxed{A = A_1 + A_2}$$

$$A_1 = \int_0^1 (\sqrt{x} - x^2)\, dx$$

$\vdots$

$$= \frac{1}{3}$$

$$A_2 = \int_1^2 (x^2 - \sqrt{x})\,dx$$

$$= \left[\frac{x^3}{3} - \frac{2}{3} x^{3/2} \right]_1^2$$

$$= \left[\frac{8}{3} - \frac{2}{3}(2^{3/2}) \right] - \left[\frac{1}{3} - \frac{2}{3} \right]$$

$$= 3 - \frac{2^{5/2}}{3}$$

$$A = A_1 + A_2$$

$$= \frac{10}{3} - \frac{2^{5/2}}{3}$$