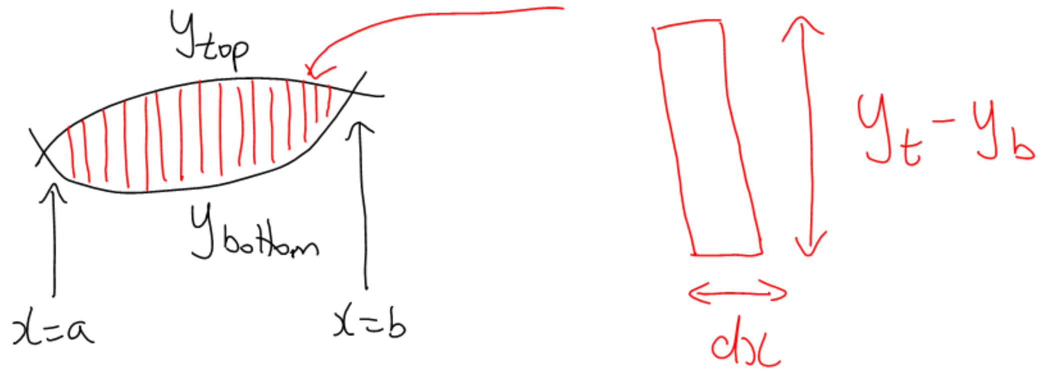
Quiz Wed Nov 15 : 25.4

26.2 Area Between 2 Curves



Vertical slices have area $\underbrace{(y_t - y_b)\,dx}_{dA}$

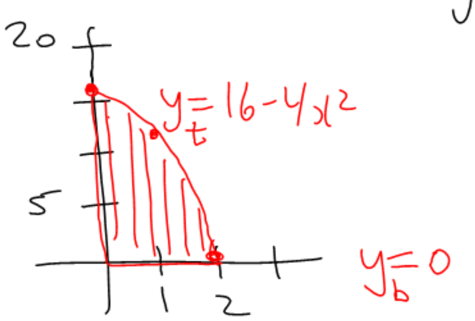
A = sum of areas of infinitely-many slices

$$= \int dA$$

$$= \int_a^b (y_t - y_b)\,dx$$

$$\boxed{A = \int_a^b (y_t - y_b)\,dx}$$

Ex: Find first-quadrant area bounded by $y = 16 - 4x^2$.



x	$y = 16 - 4x^2$
0	16
1	12
2	0

$$A = \int_0^2 (y_t - y_b)\,dx$$

$$= \int_0^2 (16 - 4x^2 - 0)\,dx$$

$$= \left[16x - \frac{4x^3}{3}\right]_0^2$$

$$= \left[32 - \frac{32}{3}\right] - [0]$$

$$= \frac{64}{3}$$

$$\boxed{\begin{aligned} \int x^2\,dx &= \underline{\underline{\frac{x^3}{3} + C}} \\ \int_0^1 x^2\,dx &= \frac{x^3}{3}\Big|_0^1 \\ &= \frac{1}{3} \end{aligned}}$$

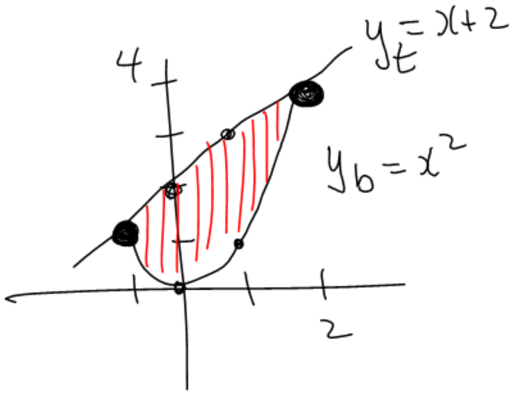
Ex: Area between $y = x^2$ and $y = x + 2$?

Intersection:

$$y = y$$
$$x^2 = x + 2$$
$$x^2 - x - 2 = 0$$
$$(x-2)(x+1) = 0$$
$$x = 2, -1$$

x	$y = x^2$
-1	1
0	0
1	1
2	4

x	$y = x + 2$
-1	1
0	2
1	3
2	4



$$A = \int_{-1}^{2} (y_t - y_b)\,dx$$

$$= \int_{-1}^{2} (x + 2 - x^2)\, dx$$

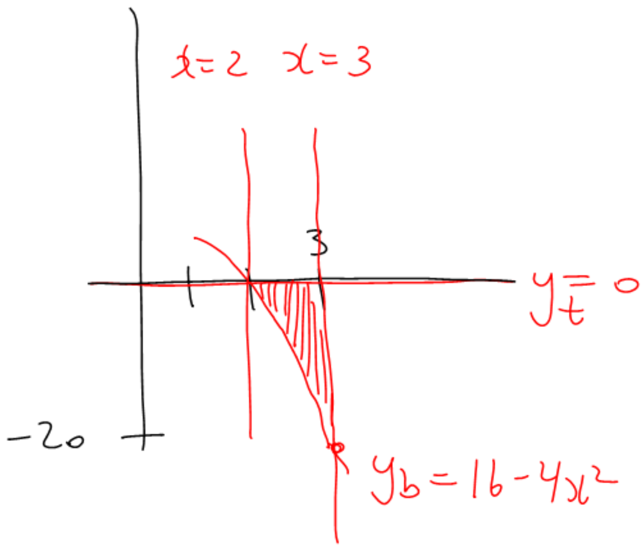
$$= \left[\frac{x^2}{2} + 2x - \frac{x^3}{3} \right]_{-1}^{2}$$

$$= \left[2 + 4 - \frac{8}{3} \right] - \left[\frac{1}{2} - 2 + \frac{1}{3} \right]$$

$$= \frac{9}{2}$$

Ex: Area bounded by $y = 16 - 4x^2$, $y = 0$, $x = 2$ and $x = 3$?

x	$y = 16 - 4x^2$
2	0
3	-20



$$A = \int_a^b (y_t - y_b)\, dx$$

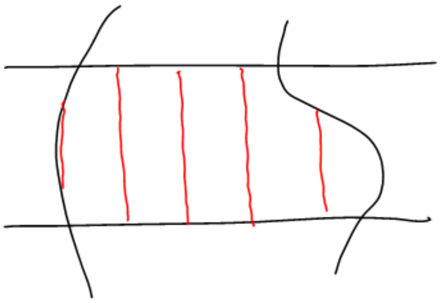
$$= \int_2^3 [0 - (16 - 4x^2)]\, dx$$

$$= \int_2^3 (-16 + 4x^2)\, dx$$

$$= \left[-16x + \frac{4x^3}{3} \right]_2^3$$

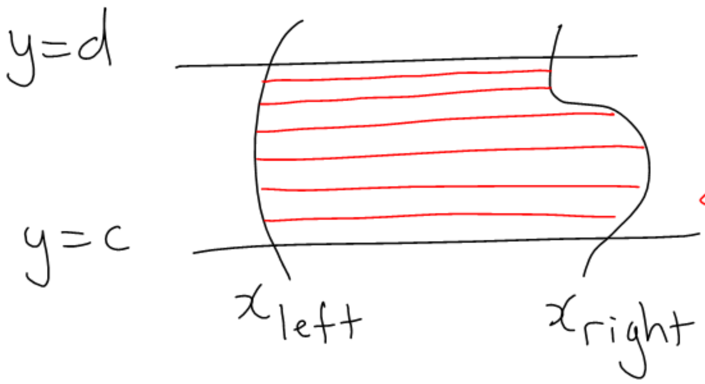
$$= \left[-48 + 36\right] - \left[-32 + \frac{32}{3}\right]$$

$$= \frac{28}{3}$$

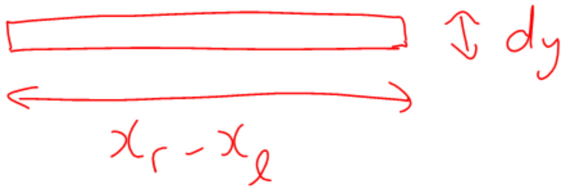


Inconvenient to use vertical slices.

y_t and y_b vary.



Use horizontal slices.



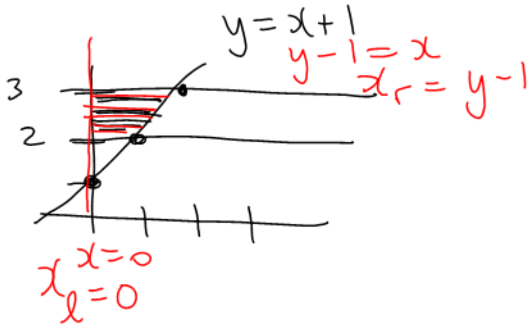
Horizontal slices have area

$$\underbrace{(x_r - x_\ell)\,dy}_{dA}$$

$$\boxed{A = \int_c^d (x_r - x_\ell)\, dy}$$

(PREVIEW)

Ex:



$$A = \int_2^3 (y-1-0)\, dy$$