26.1 Cont'd

Ex: Ball is thrown straight up from 12 m high. Takes 6 s to land. Find initial velocity v_0.

$$a(t) = -9.8$$

$$v(t) = \int -9.8\,dt$$

$$v(t) = -9.8t + C_1$$

$t = 0$: $v = v_0$:

$$v_0 = C_1$$

$$\boxed{v(t) = -9.8t + v_0}$$

$$h(t) = \int v(t)\,dt$$

$$h(t) = \int (-9.8t + v_0)\,dt$$

$$h(t) = -\frac{9.8t^2}{2} + v_0 t + C_2$$

$h = 12$, $t = 0$:

$$12 = 0 + 0 + C_2$$

$$C_2 = 12$$

$$\boxed{h(t) = -4.9t^2 + v_0 t + 12}$$

$h = 0$:
$t = 6$:

$$0 = -4.9(36) + v_0(6) + 12$$
$$164.4 = 6v_0$$
$$v_0 = 27.4 \text{ m/s}$$

Displacement: Distance travelled from the starting point.

$s(0) = 0$ (unless otherwise specified)

Ex: Car travels in a straight line with $a(t) = -4t$ m/s^2 (car is braking). Brakes are applied when velocity $= 20$ m/s. Stopping distance?

$$a(t) = -4t$$
$$v(t) = \int -4t\, dt$$
$$v(t) = -2t^2 + C_1$$

$t = 0$
brakes are applied
$v(0) = 20$

$v = 20$, $t = 0$: $20 = 0 + C_1$

$$C_1 = 20$$

$$\boxed{v(t) = -2t^2 + 20}$$

$$s(t) = \int (-2t^2 + 20)\,dt$$

$$s(t) = \frac{-2t^3}{3} + 20t + C_2$$

$s = 0$, $t = 0$:

$$0 = 0 + 0 + C_2$$

$$C_2 = 0$$

$$\boxed{s(t) = \frac{-2t^3}{3} + 20t}$$

Stopping Time

Set $v = 0$:

$$-2t^2 + 20 = 0$$

$$20 = 2t^2$$

$$10 = t^2$$

$$t = \pm\sqrt{10} \qquad t = \sqrt{10}$$

Stopping Distance

$$s(\sqrt{10}) = -\frac{2}{3}\sqrt{10}^3 + 20(\sqrt{10})$$

$$\approx 42 \text{ m}$$

Ex: Angular velocity of a rotor is $\frac{d\theta}{dt} = \sqrt{(4t+1)^3}$

Find angular displacement θ after 5 s.

$$\theta = \int \frac{d\theta}{dt}\, dt$$

$$= \int (4t+1)^{3/2}\, dt$$

$u = 4t+1$

$\frac{du}{dt} = 4$

$du = 4\,dt$

$\frac{du}{4} = dt$

$$= \frac{1}{4}\int u^{3/2}\, du$$

$$= \frac{1}{4}\left(\frac{2}{5}u^{5/2}\right) + C$$

$$= \frac{1}{4}\left(\frac{2}{5}\right)(4t+1)^{5/2} + C$$

$t = 0$:
$\theta = 0$
(displacement)

$$0 = \frac{1}{4}\left(\frac{2}{5}\right)(1)^{5/2} + C$$

$$C = -0.1$$

$$\theta(t) = 0.1\,(4t+1)^{5/2} - 0.1$$

$$\theta(5) = 0.1\,(21)^{5/2} - 0.1$$

$$\approx 202 \text{ radians}$$