25.6 Cont'd

Trapezoidal Rule and Simpson's Rule on formula sheet.

$\Delta x = \frac{b-a}{n}$ for both rules.

For Simpson's Rule, n must be <u>even</u>.

<u>Ex</u>: Approximate $\int_1^5 \sqrt{1+x^2}\,dx$ using Simpson's Rule with 4 intervals. Answer to 2 decimal places.

$\Delta x = \frac{b-a}{n} = \frac{5-1}{4} = 1$ (n is even ✓)

x	$y = \sqrt{1+x^2}$	
1	$\sqrt{2}$ (or 1.4142)	$\Leftarrow y_0$
2	$\sqrt{5}$	$\Leftarrow y_1$
3	$\sqrt{10}$	
4	$\sqrt{17}$	
5	$\sqrt{26}$	$\Leftarrow y_4$

Start at 1
End at 5
Go up by 1

$$\int_1^5 \sqrt{1+x^2}\,dx \approx \frac{\Delta x}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\right)$$

$$\approx \frac{1}{3}\left(\sqrt{2} + 4\sqrt{5} + 2\sqrt{10} + 4\sqrt{17} + \sqrt{26}\right)$$

$$\approx 12.76$$

Ex: Approximate $\int_0^2 7^x\,dx$ to 2 decimal places with $n=4$ using:

a) Trapezoidal Rule

b) Simpson's Rule

a) $\Delta x = \frac{b-a}{n} = \frac{2-0}{4} = 0.5$

x	$y=7^x$	
0	1	$\leftarrow y_0$
0.5	$7^{0.5}$	$\leftarrow y_1$
1	7	
1.5	$7^{1.5}$	
2	49	$\leftarrow y_4$

Start at 0
End at 2
Go up by 0.5

$$\int_0^2 7^x\,dx \approx \frac{\Delta x}{2}\left[y_0+2y_1+2y_2+2y_3+y_4\right]$$

$$\approx \frac{0.5}{2}\left[1+2\left(7^{0.5}+7+7^{1.5}\right)+49\right]$$

$$\approx 26.58$$

b) (n is even ✓)

$$\int_0^2 7^x\,dx \approx \frac{\Delta x}{3}\left[y_0+4y_1+2y_2+4y_3+y_4\right]$$

$$\approx \frac{0.5}{3}\left[1+4(7^{0.5})+2(7)+4(7^{1.5})+49\right]$$

$$\approx 24.78$$

For reference $\int_0^2 7^x\,dx \approx 24.67$

Simpson's Rule is generally much better than Trapezoidal.

ASIDE $\int 7^x\,dx = \frac{7^x}{\ln 7} + C$ (See in Math193)

$$\int_0^2 7^x\,dx = \left.\frac{7^x}{\ln 7}\right|_0^2$$

$$= \frac{49}{\ln 7} - \frac{1}{\ln 7}$$

26.1 Applications of Integration

(Displacement, Velocity, Acceleration)

Recall $s(t)$ or $h(t)$ = displacement

$v(t)$ = velocity

$a(t)$ = acceleration

$$v(t) = s'(t)$$
$$a(t) = v'(t) = s''(t)$$

$$v(t) = \int a(t)\,dt$$
$$s(t) = \int v(t)\,dt$$

Ex: A ball is thrown straight up from the ground with initial velocity 3 m/s. Find height $h(t)$.

$a(t) = -9.8$ (gravity)

$v(t) = \int (-9.8)\,dt$

$$\boxed{v(t) = -9.8t + C_1}$$

Sub $t=0$, $v=3$: $3 = 0 + C_1$

$C_1 = 3$

$$\boxed{v(t) = -9.8t + 3}$$

$$h(t) = \int v(t)\,dt$$

$$h(t) = \int (-9.8t + 3)\,dt$$

$$\boxed{h(t) = -\frac{9.8t^2}{2} + 3t + C_2}$$

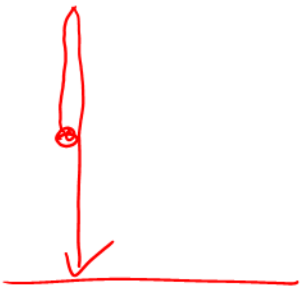
$t = 0$
$h = 0$:

$$0 = 0 + 0 + C_2$$

$$C_2 = 0$$

$$\boxed{h(t) = -4.9t^2 + 3t}$$

Ex: Ball is thrown straight up from 12m high. Takes 6 s to land. Find the initial velocity v_0.



$$a(t) = -9.8 \quad \text{(gravity)}$$