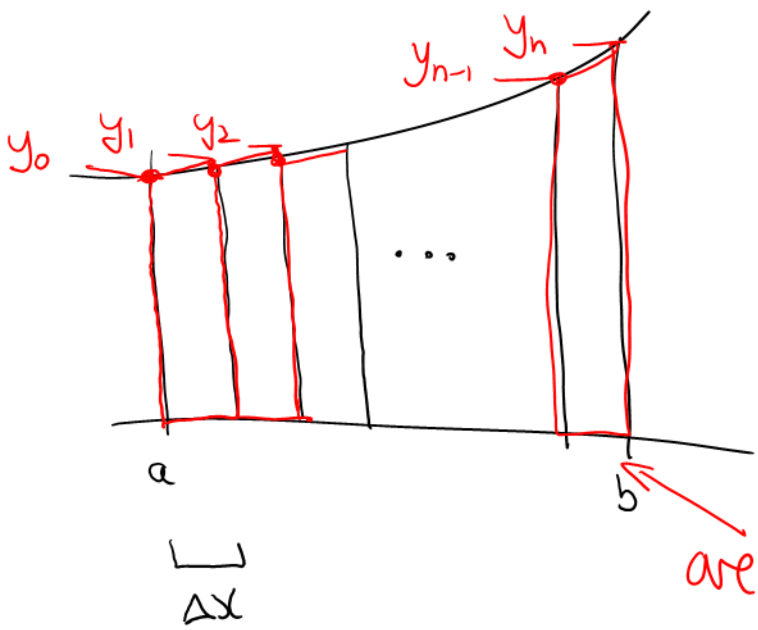
Quiz Wed Section 25.2

25.5 The Trapezoidal Rule

Want to approximate $\int_a^b f(x)\,dx$.

Divide into n intervals and approximate by trapezoids



area of trapezoid

$$= \text{base}\left(\frac{\text{height}_1 + \text{height}_2}{2}\right)$$

$$= \Delta x\left(\frac{y_{n-1} + y_n}{2}\right)$$

$$= \frac{\Delta x}{2}\left(y_{n-1} + y_n\right)$$

$$\text{Approx. area} = \frac{\Delta x}{2}(y_0 + y_1) + \frac{\Delta x}{2}(y_1 + y_2) + \dots + \frac{\Delta x}{2}(y_{n-1} + y_n)$$

$$= \frac{\Delta x}{2}\left[y_0 + \underline{y_1 + y_1} + \underline{y_2 + \dots} + \underline{y_{n-1}} + y_n\right]$$

$$= \frac{\Delta x}{2}\left[y_0 + 2y_1 + 2y_2 + \dots + 2y_{n-1} + y_n\right]$$

$$= \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n)\right]$$

Trapezoidal Rule

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + \dots + 2f(x_{n-1}) + f(x_n)\right]$$

where $\Delta x = \frac{b-a}{n}$

Ex: Approximate using the trapezoidal rule with 4 intervals. Answer to 4 decimal places.

$$\int_2^3 \frac{1}{x}\,dx.$$

$$\Delta x = \frac{b-a}{n} = \frac{3-2}{4} = 0.25$$

x	$y = \frac{1}{x}$	
2	0.5	$\leftarrow y_0$
2.25	0.444444	
2.5	0.4	
2.75	0.363636	
3	0.333333	$\leftarrow y_4$

(To 6 decimal places)

Start at 2
End at 3
Go up by 0.25

$$\int_2^3 \frac{1}{x}\,dx \approx \frac{\Delta x}{2}\left[y_0 + 2y_1 + 2y_2 + 2y_3 + y_4\right]$$

$$\approx \frac{0.25}{2}\left[0.5 + 2(0.444444 + 0.4 + 0.363636) + 0.333333\right]$$

$$\approx 0.4062$$

As $n \to \infty$, the approximation gets better.

Ex: Approximate using the trapezoidal rule with 5 intervals. Answer to 2 decimal places.

$$\int_0^2 3^x\,dx$$

$$\Delta x = \frac{b-a}{n} = \frac{2-0}{5} = 0.4$$

x	$y = 3^x$	
0	1	$\leftarrow y_0$
0.4	$3^{0.4}$	
0.8	$3^{0.8}$	
1.2	$3^{1.2}$	
1.6	$3^{1.6}$	
2	9	$\leftarrow y_5$

Start at 0
End at 2
Go up by 0.4

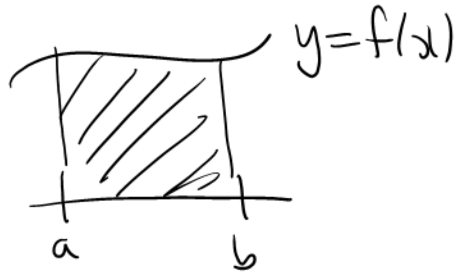
$$\int_0^2 3^x\,dx \approx \frac{\Delta x}{2}\left[y_0 + 2(y_1 + \dots + y_4) + y_5\right]$$

$$\approx \frac{0.4}{2}\left[1 + 2\left(3^{0.4} + 3^{0.8} + 3^{1.2} + 3^{1.6}\right) + 9\right]$$

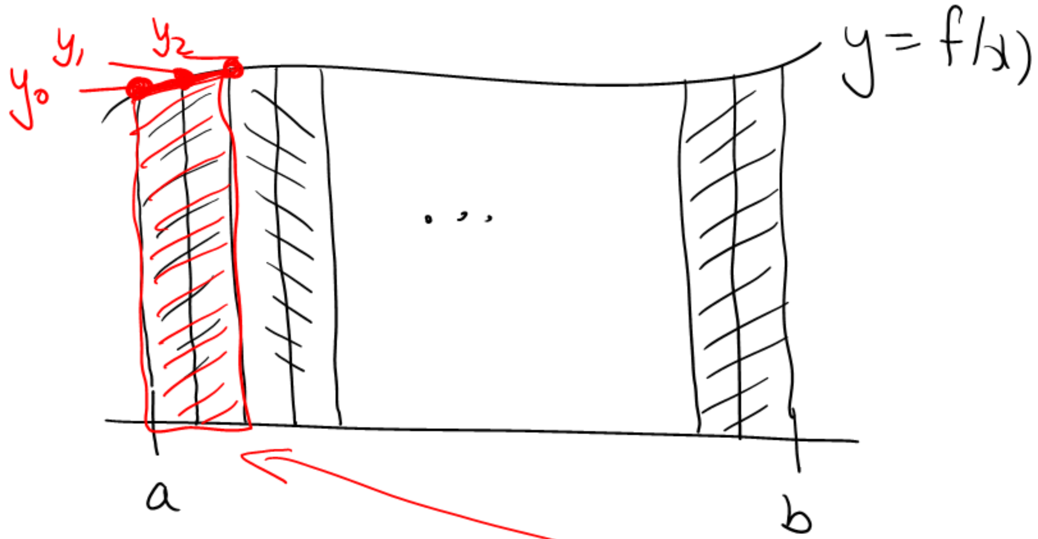
$$\approx 7.40$$

25.6 Simpson's Rule

Want to approximate $\int_a^b f(x)\,dx$.



Divide into n intervals, where n is <u>even</u>.



Δx

$2\Delta x$

approximate area

= base (approximate height)

$$= 2\Delta x\left(\frac{y_0 + 4y_1 + y_2}{6}\right)$$

$$= \frac{\Delta x}{3}\left(y_0 + 4y_1 + y_2\right)$$

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}\left[\left(y_0+4y_1+y_2\right) + \underline{\underline{\left(y_2+4y_3+y_4\right)}} + \dots\right]$$

$$\approx \frac{\Delta x}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \dots + 4y_{n-1} + y_n\right]$$

Simpson's Rule

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + \dots + 4f(x_{n-1}) + f(x_n)\right]$$

where $\Delta x = \frac{b-a}{n}$

and <u>n is even</u>

Coefficients: 1 - 4 - 2 - 4 - 2 - ... - 4 - 1