Fundamental Theorem of Line Integrals

$$\boxed{\int_C \nabla f \cdot d\vec{r} = f(B) - f(A)}$$

$f(B)$, $f(A)$: endpoints of C

Generalization of $\int_a^b f'(x)\,dx = f(x)\Big|_a^b$

Ex: Evaluate $\int_C \left[y\,dx + (x - z\sin y)\,dy + \cos y\,dz\right]$

where $C: (3,0,1) \to (4,\pi,5)$

Recall $d\vec{r} = [dx, dy, dz]$

$$\text{Integral} = \int_C \underbrace{[y,\ x - z\sin y,\ \cos y]}_{\vec{F}} \cdot \underbrace{[dx, dy, dz]}_{d\vec{r}}$$

Is $\vec{F} = \nabla f$?

$$\operatorname{curl}\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & x - z\sin y & \cos y \end{vmatrix} = \vec{0}$$

Yes

$f = \int y\,dx$ AND $f = \int (x - z\sin y)\,dy$ AND $f = \int \cos y\,dz$

$$\Rightarrow \quad f = xy + z\cos y$$

$$\text{Integral} = \int_C \nabla f \cdot d\vec{r}$$

$$= f\Big|_A^B$$

$$= xy + z\cos y \Big|_{(3,0,1)}^{(4,\pi,5)}$$

$$= 4\pi + \underbrace{\cancel{5\cos\pi}}_{-5} - (0 + 1)$$

$$= 4\pi - 6$$

$$\int_C \vec{F} \cdot d\vec{r}$$

$\vec{F} = \nabla f$

Fund. Thm. of Line Integrals

$\vec{F} \neq \nabla f$

Parametrize curve