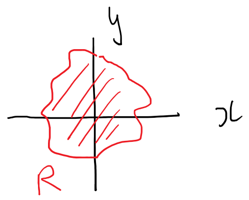
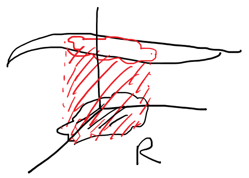
13.3 Area and Volume by Double Integration



$$A = \iint_R dA$$

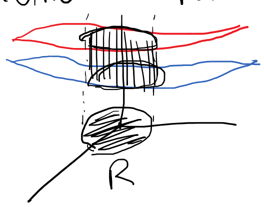
dA: $dy\,dx$ or $dx\,dy$

Volume under $z=f$, over R



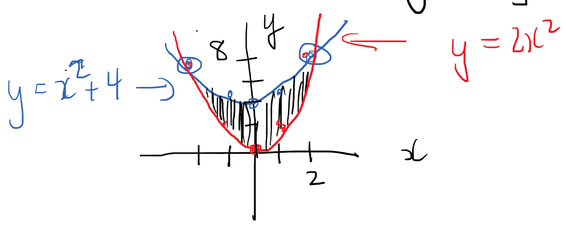
$$V = \iint_R f\, dA$$

Volume between $z=z_1$ and $z=z_2$, over R



$$V = \iint_R (z_2 - z_1)\, dA$$

Ex: Set up a double integral for the area bounded by $y = 2x^2$ and $y = x^2+4$



Intersection
$$y = y$$
$$2x^2 = x^2 + 4$$
$$x^2 = 4$$
$$x = \pm 2$$

$$\boxed{\begin{aligned} R:\quad 2x^2 &\le y \le x^2+4 \\ -2 &\le x \le 2 \end{aligned}}$$

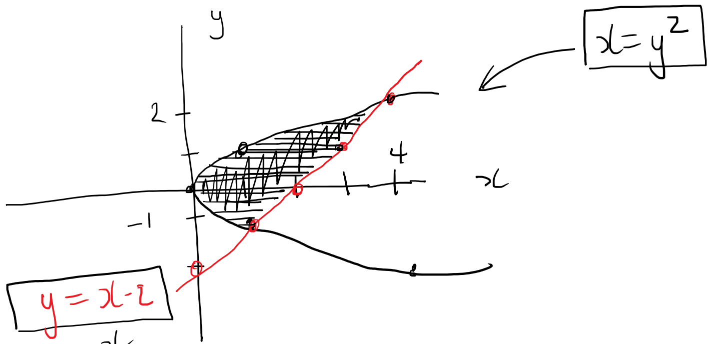
$$A = \iint_R dy\,dx$$

$$A = \int_{-2}^{2} \int_{2x^2}^{x^2+4} dy \; dx$$

Ex: Set up a double integral for the volume between $z=6$ and $z=x+1$, over the region bounded by $x=y^2$ and $y=x-2$

$x=y^2$

x	y
0	0
1	± 1
4	± 2



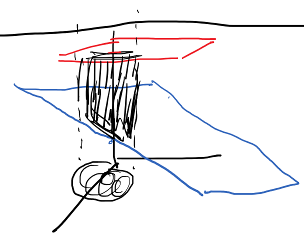
$y+2=x$

$x=y+2$

$$\boxed{\begin{aligned} y^2 \le\ & x \le y+2 \\ -1 \le\ & y \le 2 \end{aligned}}$$

$\boxed{6} > \boxed{x+1}$

z_{top} $\quad$ z_{bottom}



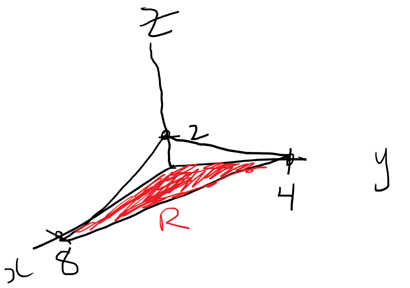
$$V = \iint_R (z_{top} - z_{bottom})\, dA$$

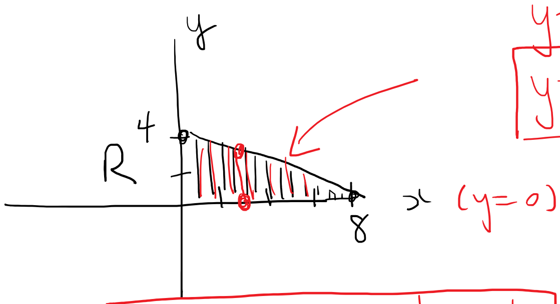
$$= \iint_R (5-x)\, dx\, dy$$

$$= \int_{-1}^{2} \int_{y^2}^{y+2} (5-x)\, dx\, dy$$

Ex: Set up a double integral for the first-octant volume under $x+2y+4z=8$

$x, y, z \geq 0$





$y = mx + b$

$$\boxed{y = -\frac{1}{2}x + 4}$$

$$\boxed{\begin{aligned} R:\; & 0 \le y \le -\tfrac{1}{2}x + 4 \\ & 0 \le x \le 8 \end{aligned}}$$

$$\boxed{\begin{aligned} x + 2y + 4z &= 8 \\ 4z &= 8 - x - 2y \\ z &= 2 - \frac{x}{4} - \frac{y}{2} \end{aligned}}$$

$$V = \iint_R z \, dA$$

$$= \int_0^8 \int_0^{-\frac{x}{2}+4} \left(2 - \frac{x}{4} - \frac{y}{2}\right) dy \, dx$$