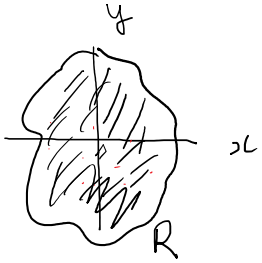
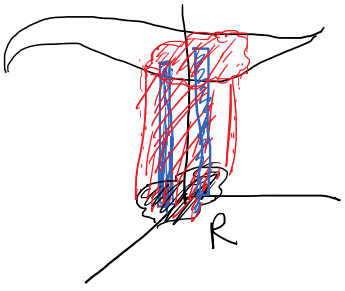
13.2 Setting Up Double Integrals

The Meaning of Double Integrals



$$\text{area} = \iint_R dA$$

$dA = dx\,dy$ or $dy\,dx$



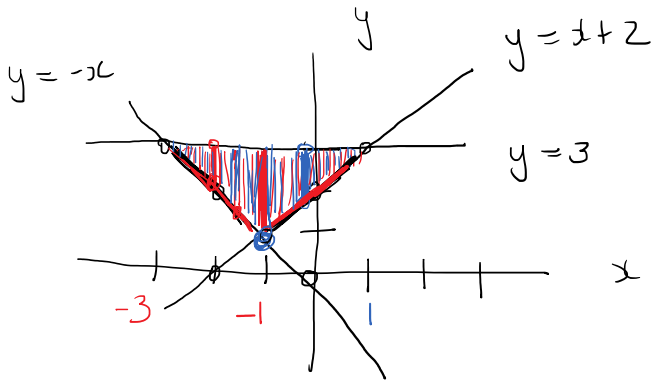
$$\text{Volume} = \iint_R f\,dA$$

Set up $\iint_R f \, dA$ ← $dx\,dy$ or $dy\,dx$

Ex: Set up $\iint_R f(x,y)\,dA$ where $f(x,y) = x$ and R is bounded by $y = -x$, $y = x+2$ and $y = 3$

Use:

a) vertical slices



Intersection
$y = y$
$-x = x+2$
$x = -1$

$-x \le y \le 3$

$-3 \le x \le -1$

and

$x+2 \le y \le 3$

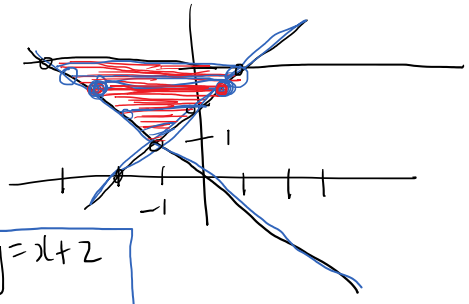
$-1 \le x \le 1$

$dA = dy\,dx$

$$\iint_R f\,dA = \iint_R x\,dy\,dx$$

$$= \int_{-3}^{-1}\int_{-x}^{3} x\,dy\,dx + \int_{-1}^{1}\int_{x+2}^{3} x\,dy\,dx$$

b) horizontal slices



$y = 3$

$y = x + 2$

$x = y - 2$

$y = -x$

$x = -y$

$$-y \leq x \leq y - 2$$

$$1 \leq y \leq 3$$

$$dA = dx\,dy$$

$$\text{Double integral} = \int_{1}^{3} \int_{-y}^{y-2} x \, dx \, dy$$

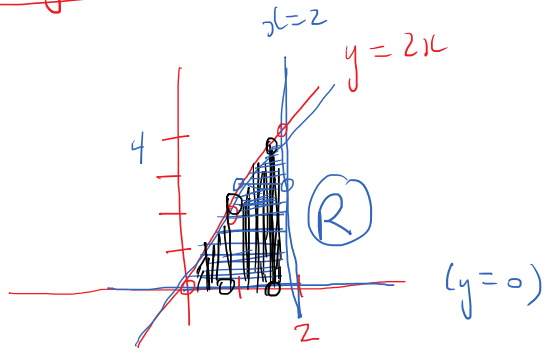
Ex: Evaluate $\int_0^4 \int_{y/2}^2 y e^{x^3} \, dx \, dy$

Can't integrate e^{x^3} w.r.t. x

Sketch region R and swap order of integration

$x = \frac{y}{2}$

$y = 2x$



Hor. Slices

$\frac{y}{2} \leq x \leq 2$

$0 \leq y \leq 4$

Vertical Slices

R: $0 \leq y \leq 2x$

$0 \leq x \leq 2$

$$\int_0^4 \int_{\frac{y}{2}}^2 y e^{x^3} \, dx \, dy = \int_0^2 \int_0^{2x} y e^{x^3} \, dy \, dx$$

$$= \int_0^2 \frac{y^2}{2} (e^{x^3}) \Big|_{y=0}^{y=2x} \, dx$$

$$= \int_0^2 2x^2 (e^{x^3})\, dx$$

$$u = x^3$$
$$du = 3x^2\, dx$$
$$\frac{2}{3}\, du = 2x^2\, dx$$

$$= \int_{x=0}^{x=2} \frac{2}{3} e^u\, du$$

$$= \frac{2}{3} e^u \Big|_{x=0}^{x=2}$$

$$= \frac{2}{3} e^{x^3} \Big|_0^2$$

$$= \frac{2}{3}\left(e^8 - 1\right)$$