

13.1 Double Integration

Double Integration

Ex: Evaluate $\int_1^2 \int_0^x (y^2 + xy^3) dy dx$

Do the inside integral first
dy: integrate with respect to y
(treat x as a constant)

$$\begin{aligned} & \int_1^2 \int_0^x (y^2 + xy^3) dy dx \\ &= \int_1^2 \left[\frac{y^3}{3} + \frac{xy^4}{4} \right]_{y=0}^{y=x} dx \\ &= \int_1^2 \left(\frac{x^3}{3} + \frac{x^5}{4} - 0 \right) dx \\ &= \left[\frac{x^4}{12} + \frac{x^6}{24} \right]_1^2 \\ &= \left(\frac{16}{12} + \frac{64}{24} \right) - \left(\frac{1}{12} + \frac{1}{24} \right) \\ &= \frac{31}{8} \end{aligned}$$