FACTS

1) $\nabla f$ points in the direction of the maximum rate of increase of $f$
2) the maximum rate of increase of $f$ is $\|\nabla f\|$

Why? 1) $D_{\vec{u}} f = \nabla f \cdot \vec{u}$

$= \|\nabla f\| \, \|\vec{u}\| \cos\theta$


Maximized when $\vec{u}$ is parallel to $\nabla f$

2) Suppose $\vec{u}$ is parallel to $\nabla f$

$$\vec{u} = \frac{\nabla f}{\|\nabla f\|}$$

$$D_{\vec{u}} f = \nabla f \cdot \vec{u}$$

$$= \nabla f \cdot \frac{\nabla f}{\|\nabla f\|}$$

$$= \frac{\|\nabla f\|^2}{\|\nabla f\|}$$

$$= \|\nabla f\|$$

Ex: $f = x^3 + y^2 + 4z$ (°C)

Point $P = (1,2,3)$ ($x, y, z$ in m)

a) From $P$, in which direction does $f$ increase fastest?

$\nabla f = [\, 3x^2 , 2y , 4 \,]$

$\nabla f(P) = [\, 3 , 4 , 4 \,]$

b) From $P$, what is the maximum rate of increase of $f$?

$$
\begin{aligned}
&\| \nabla f(P) \| \\
&= \| [3, 4, 4] \| \\
&= \sqrt{9 + 16 + 16} \\
&= \sqrt{41} \quad \frac{^{\circ}C}{m}
\end{aligned}
$$