

Absolute Error

Ex: The range of a projectile is

$$R = \frac{v_0^2 \sin 2\theta}{9.8}$$

$$\text{Given } \theta = 30^\circ \pm 1^\circ$$

$$v_0 = 3 \pm 0.2 \text{ m/s,}$$

find the maximum error in the range.

$$df = f_x dx + f_y dy$$

$$dR = R_{v_0} dv_0 + R_\theta d\theta$$

$$dR = \frac{2v_0 \sin 2\theta}{9.8} dv_0 + \frac{2v_0^2 \cos 2\theta}{9.8} d\theta$$

$$\theta = 30^\circ \quad v_0 = 3$$

Goal: Maximize dR

$$\Rightarrow \text{Use } d\theta = 1^\circ = \frac{\pi}{180} \text{ rads} \quad dv_0 = 0.2$$

$$dR_{\max} = \frac{6 \sin 60^\circ}{9.8} (0.2) + \frac{18 \cos 60^\circ}{9.8} \left(\frac{\pi}{180} \right)$$

$$\approx 0.1 \text{ m}$$

$$\Rightarrow \Delta R_{\max} \approx 0.1 \text{ m}$$

$$\Rightarrow \text{Max error in range is } \pm 0.1 \text{ m}$$