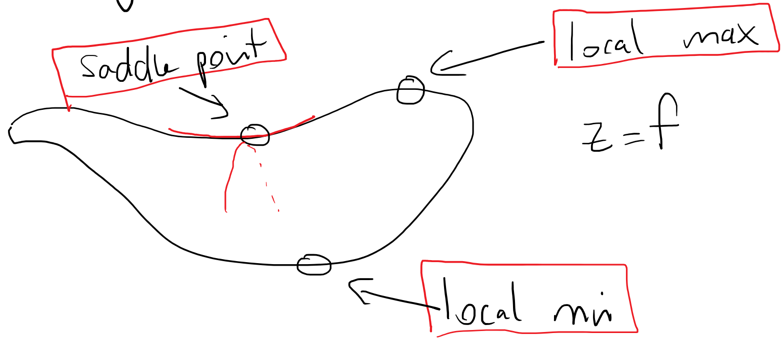
12.10 Classifying Critical Points

Classifying Critical Points

Recall: A <u>critical point</u> of f is a point where f_x and f_y are 0 or undefined



Let $\Delta = f_{xx} f_{yy} - (f_{xy})^2$

2nd Derivative Test

If	then
$\Delta > 0$ and $f_{xx} > 0$	local min
$\Delta > 0$ and $f_{xx} < 0$	local max
$\Delta < 0$	saddle point
$\Delta = 0$	no info

Ex. Find and classify all critical points

$$f = 4x - \frac{x^3}{3} - xy^2$$

$$\left.\begin{array}{l} ① \quad f_x = 4 - x^2 - y^2 = 0 \\ ② \quad \boxed{f_y = -2xy = 0} \end{array}\right\} \quad 0 \text{ or undefined}$$

$x=0$

$\to ①: \quad 4-y^2=0$

$y^2=4$

$y=\pm 2$

$(0,\pm 2)$

or

$y=0$

$\to ①: \quad 4-x^2=0$

$\vdots$

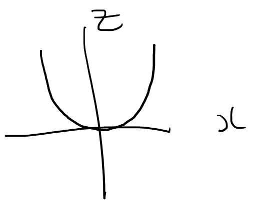
$x=\pm 2$

$(\pm 2, 0)$

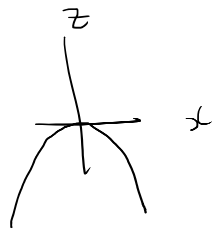
Points	$f_{xx} = -2x$	$f_{yy} = -2x$	$f_{xy} = -2y$	$\Delta = f_{xx}f_{yy} - (f_{xy})^2$	Conclude
$(0,2)$	0	0	-4	-16	Saddle Point
$(0,-2)$	0	0	4	-16	Saddle Point
$(2,0)$	-4	-4	0	16	Local Max
$(-2,0)$	4	4	0	16	Local Min

Geometry Explanation:

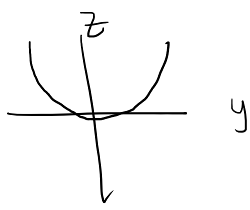
$f_{xx} > 0$



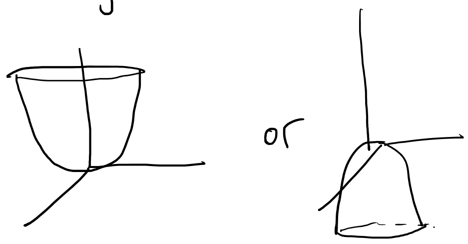
$f_{xx} < 0$



$f_{yy} > 0$



$f_{xx} f_{yy} > 0 \implies$



Sign of f_{xx} determines which one

$|f_{xy}|$ large $\implies$ f twists