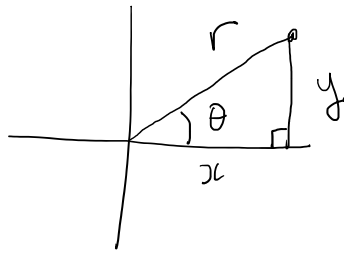
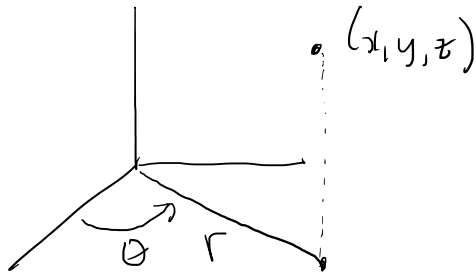


11.8 Cylindrical and Spherical Coordinates

Cylindrical Coordinates

Rectangular coordinates (x, y, z)

Cylindrical coordinates (r, θ, z)



$$x = r \cos \theta \quad y = r \sin \theta$$

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1} \left(\frac{y}{x} \right) \quad (+\pi?)$$

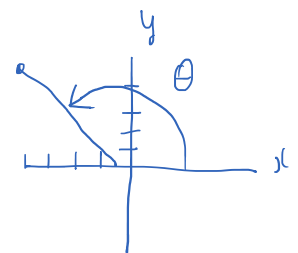
Ex: Convert $(x, y, z) = (-4, 4, 5)$ to cylindrical
 $z = 5$

$$r = \sqrt{x^2 + y^2} = \sqrt{32} = 4\sqrt{2}$$

$$\theta = \tan^{-1}(-1) \quad (+\pi?)$$

$$= -\frac{\pi}{4} + \pi$$

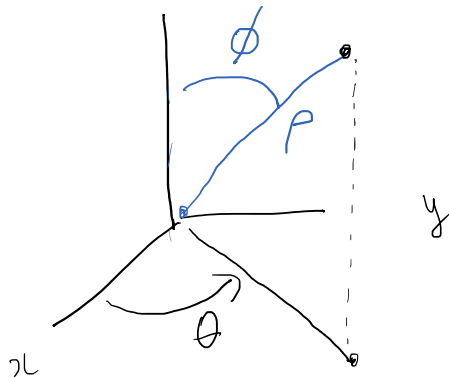
$$= \frac{3\pi}{4}$$



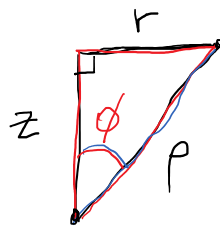
$$(r, \theta, z) = (4\sqrt{2}, \frac{3\pi}{4}, 5)$$

Spherical Coordinates

Spherical Coordinates (ρ, ϕ, θ)



$$\begin{aligned} \rho &\geq 0 \\ 0 &\leq \phi \leq \pi \\ 0 &\leq \theta < 2\pi \end{aligned}$$



$$\begin{aligned} \rho^2 &= z^2 + r^2 \\ z &= \rho \cos \phi \\ r &= \rho \sin \phi \\ \rho &= \sqrt{x^2 + y^2 + z^2} \end{aligned}$$

Ex: Express in spherical

a) $x^2 + y^2 + z^2 = 9$

$$\rho^2 = 9$$

$$b) \quad z = 2 \sqrt{x^2 + y^2}$$

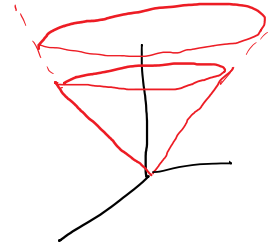
$$z = 2r$$

$$\rho \cos \phi = 2 \rho \sin \phi$$

$$\cos \phi = 2 \sin \phi$$

$$\frac{1}{2} = \tan \phi$$

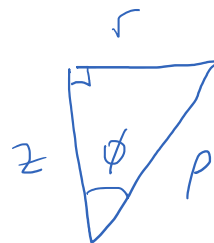
$$\phi = \tan^{-1} \frac{1}{2}$$



$$c) \quad z = 3$$

$$\rho \cos \phi = 3$$

$$\text{or } \rho = 3 \sec \phi$$



Ex: Express in rectangular:

$$\rho = \csc \phi \cot \phi$$

$$\rho = \frac{1}{\sin \phi} \frac{\cos \phi}{\sin \phi}$$

$$\rho \sin^2 \phi = \cos \phi$$

$$\rho^2 \sin^2 \phi = \rho \cos \phi$$

$$r^2 = z$$

$$z = x^2 + y^2$$

