1. [3 marks] A city has an average of 1.9 power failures per year. Find the probability that the city has at most three power failures over the next two years. Round your answer to two decimal places.

Poisson $\mu = 1.9$ failures/year $= 3.8$ failures/2 years

$\boxed{\text{Use } \mu = 3.8}$ $X = \#$ failures

$$P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$$

$$= e^{-3.8}\left(\frac{3.8^0}{0!} + \frac{3.8^1}{1!} + \frac{3.8^2}{2!} + \frac{3.8^3}{3!}\right)$$

$$\approx 0.47$$

2. [3 marks] A drilling company has a 23% probability of success on each drilling attempt. Find the probability that at least 3 of their next 11 attempts are successful. Round your answer to two decimal places.

Binomial $p = 0.23$ $n = 11$

$q = 1 - p = 0.77$

$X = \#$ successes

$$P(X \geq 3) = 1 - P(X=0) - P(X=1) - P(X=2)$$

$$= 1 - {}_{11}C_0 (0.23)^0 (0.77)^{11}$$

$$- {}_{11}C_1 (0.23)^1 (0.77)^{10} - {}_{11}C_2 (0.23)^2 (0.77)^9$$

$$\approx 0.48$$

3. [4 marks] Without any estimate for the population proportion p, what is the minimum sample size that guarantees a 99% margin of error for p of less than 0.003?

$$99\% \ M/E < 0.003$$

$$z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} < 0.003$$

$$\boxed{\hat{p} \text{ unknown: use } \hat{p} = 0.5 = \hat{q}}$$

$$\frac{2.576\sqrt{0.5 \times 0.5}}{\sqrt{n}} < 0.003$$

$$\frac{2.576\,(0.5)}{0.003} < \sqrt{n}$$

Square both sides:

$$184327.T < n$$

$$\boxed{\text{Minimum sample size is } n = 184328}$$

4. [3 marks] At an engineering firm, employees work an average of 46 hours per week with a SD of 4 hours per week. Find the probability that 35 randomly selected employees worked a total of less than 1568 hours last week.

$\mu = 46$, $\sigma = 4$, $n = 35$

$$P(\text{total} < 1568)$$

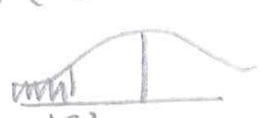
$$\left\{ \begin{array}{l} \text{total} = 1568 \\ \frac{\text{total}}{35} = \frac{1568}{35} \\ \bar{x} = 44.8 \end{array} \right.$$

$$= P(\bar{x} < 44.8)$$

$(n \geq 30 \checkmark)$

$$= P(z < -1.77)$$

$$\left\{ \begin{array}{l} z = \frac{\bar{x} - \mu}{(\sigma/\sqrt{n})} \\ = \frac{44.8 - 46}{(4/\sqrt{35})} \\ \approx -1.77 \end{array} \right.$$

$=$ 

-1.77

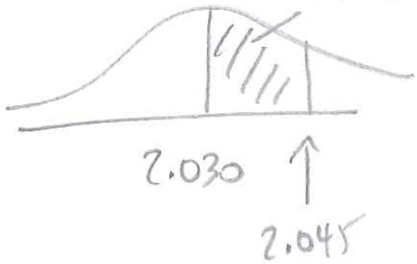
$$= 0.5 - 0.4616$$

$$= 0.0384$$

5. [3 marks] The volume in bottles of Canada Dry Ginger Ale is normally distributed with a mean of 2.030L. Given that 24.22% of volumes are between 2.030L and 2.045L, find σ. Round your answer to two decimal places.

0.2422

$\mu = 2.030$



Reverse look-up area 0.2422 :

$z = 0.65$

Sub $\mu = 2.030$ $X = 2.045$ $z = 0.65$

$$\rightarrow \quad z = \frac{X - \mu}{\sigma}$$

$$0.65 = \frac{2.045 - 2.030}{\sigma}$$

$$0.65 = \frac{0.015}{\sigma}$$

$$\sigma = \frac{0.015}{0.65}$$

$$\sigma \simeq 0.02 \text{ L}$$

6. [3 marks] The random variable X has probability density function

$$f(x) = \begin{cases} \frac{1}{8\sqrt{x}}, & 1 \leq x \leq 25 \\ 0, & \text{otherwise} \end{cases}$$

Find the value of c so that $P(4 \leq X \leq c) = 0.16$.

$$P(4 \leq X \leq c) = \int_4^c f(x)\,dx \quad 1$$

$$= \int_4^c \frac{1}{8\sqrt{x}}\,dx$$

$$= \left(\frac{1}{4} x^{1/2}\right]_4^c$$

$$= \frac{1}{4}\sqrt{c} - \frac{1}{4}\sqrt{4}$$

$$= \frac{1}{4}\sqrt{c} - \frac{1}{2} \quad 1$$

Now set $\frac{1}{4}\sqrt{c} - \frac{1}{2} = 0.16$

$$\frac{1}{4}\sqrt{c} = 0.66$$

$$\sqrt{c} = 2.64 \quad 1$$

$$c = 6.9696$$

7. [6 marks] Test whether μ_1 is less than μ_2 at the 1% significance level given the following sample data:
$n_1 = 60, \overline{x}_1 = 78.1, s_1 = 2.4, n_2 = 50, \overline{x}_2 = 79.4, s_2 = 3.5$.

a) State H_0 and H_a

LEFT-TAILED TEST

2 $H_0: \mu_1 = \mu_2$ OR $\mu_1 - \mu_2 = 0$ $H_a: \mu_1 < \mu_2$ OR $\mu_1 - \mu_2 < 0$

b) Check any necessary assumptions.

1 $n_1, n_2 \geq 30$

c) Do you reject H_0? Show all your work.

$$z = \frac{\bar{x}_1 - \bar{x}_2 - D_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \leftarrow = 0 \text{ from } H_0$$

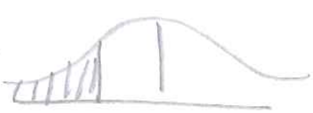
$$= \frac{78.1 - 79.4 - 0}{\sqrt{2.4^2/60 + 3.5^2/50}}$$

1 $\simeq -2.23$

1

$\boxed{\text{Don't reject } H_0 \quad \mu_1 \approx \mu_2}$

d) Find the p-value.

P= 

$= 0.5 - 0.4871$

1 $= 0.0129$