

Name: \_\_\_\_\_

Marks may be deducted for not showing all your work.

1. [3 marks] A fair die is rolled 20 times. What is the probability of rolling a 1 four, five or six times? Round your answer to 2 decimal places.

$$\text{Binomial } n=20 \quad p=\frac{1}{6} \quad q=1-p=\frac{5}{6}$$

↑  
P(roll a 1)

$$P(4, 5, \text{ or } 6 \text{ successes}) = 20C4 \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^{16} + 20C5 \left(\frac{1}{6}\right)^5 \left(\frac{5}{6}\right)^{15} + 20C6 \left(\frac{1}{6}\right)^6 \left(\frac{5}{6}\right)^{14}$$

$$\approx 0.40$$

I also accepted 0.39

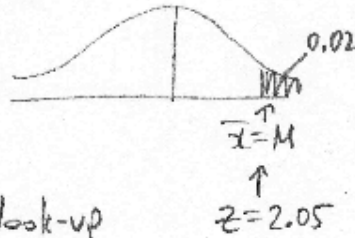
2. [3 marks] Each week there are an average of 4 accidents on a certain stretch of highway. What is the probability of observing at most three accidents on that stretch next week? Round your answer to 2 decimal places.

$$\text{Poisson } \mu=4 \quad P(X \leq 3) = \left( \frac{4^0}{0!} + \frac{4^1}{1!} + \frac{4^2}{2!} + \frac{4^3}{3!} \right) e^{-4}$$

$$\approx 0.43$$

3. [5 marks] The weight of adult females in Canada is normally distributed with mean 130 lbs and SD  $\sigma = 25$  lbs.

a) Given 10 randomly-chosen adult females in Canada; Find a value  $M$  so that the probability that their weights average to more than  $M$  is 0.02. Round your answer to 2 decimal places.



or we formula sheet  $z = 2.054$

Reverse look-up  
By Central Limit Theorem,

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$\begin{aligned} \mu &= 130 \\ \sigma &= 25 \\ n &= 10 \end{aligned}$$

$$2.05 = \frac{M - 130}{25/\sqrt{10}}$$

$$2.05 \times \frac{25}{\sqrt{10}} = M - 130$$

$$M = 2.05 \times \frac{25}{\sqrt{10}} + 130 \approx 146.21 \text{ lbs.}$$

b) What assumption(s) do you have to check?

The population is normally distributed.

4. [5 marks] A sample of 1500 ball bearings shows that 40 are defective.

a) Find a 95% upper confidence bound for the proportion defective. Round your answer to 3 decimal places.

$$\hat{p} = \frac{40}{1500} \quad n = 1500 \quad \hat{q} = 1 - \hat{p} = \frac{1460}{1500}$$

$$p \leq \hat{p} + z_{\alpha} \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$z_{\alpha} = 1.645$$

$$p \leq \frac{40}{1500} + 1.645 \sqrt{\frac{(40 \times 1460)}{1500^2}}$$

$$p \leq 0.034$$

↑  
-0.5 if you write  $\hat{p}$

I forgot you  
if you rounded  
to 2 decimal places.

b) Would a 90% upper confidence bound be larger or smaller than your 95% upper confidence bound from part a)? Explain.

We are less certain that the 90% UCB  
is at least as big as  $p$ ,

so the 90% UCB is smaller than 0.034

Alternatively: just calculate  
the 90% UCB.

5. [3 marks] We want to estimate the population proportion  $p$  with a 99% margin of error of at most 0.02. We don't have any estimates for  $\hat{p}$ . What is the minimum sample size we can use?

$$99\% \text{ M/E} \quad z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}} \leq 0.02$$

$\hat{p}\hat{q}$  can be as big  
as  $0.5 \times 0.5$

$$2.576 \sqrt{\frac{0.5 \times 0.5}{n}} \leq 0.02$$

Use this when  
 $\hat{p}$  is unknown.

$$\frac{2.576 \sqrt{0.5 \times 0.5}}{0.02} \leq \sqrt{n}$$

$$\left[ \frac{2.576 \sqrt{0.5 \times 0.5}}{0.02} \right]^2 \leq n$$

$$n \geq 4147.36$$

$$n \geq 4148$$

Minimum sample size  
we can use is 4148.

6. [6 marks] You are given the following data:  $\bar{x}_1 = 18.92$ ,  $s_1 = 4.25$ ,  $n_1 = 50$ ;  $\bar{x}_2 = 21.30$ ,  $s_2 = 3.75$ ,  $n_2 = 40$ . Test at  $\alpha = 0.01$  whether there is a difference between the two population means.

a) State  $H_0$  and  $H_a$

$$H_0: \mu_1 - \mu_2 = 0 \quad H_a: \mu_1 - \mu_2 \neq 0$$

b) What assumption(s) do you need to check?

$$n_1 \geq 30 \checkmark$$

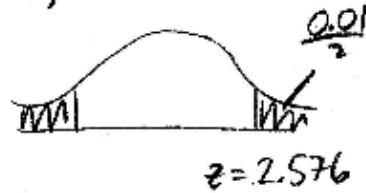
$$n_2 \geq 30 \checkmark$$

c) Do you reject  $H_0$  or not? Show all your work.

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$\approx -2.82$$

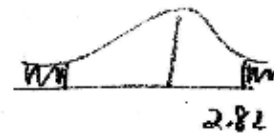
$$(D_0 = 0)$$



Reject  $H_0$ .

d) Find the p-value of the test.

$$\begin{aligned} p &= P(|z| > 2.82) \\ &= 2 \times (0.5 - 0.4976) \\ &= 0.0048 \end{aligned}$$



Name: \_\_\_\_\_

Marks may be deducted for not showing all your work.

1. [3 marks] A fair die is rolled 30 times. What is the probability of rolling a 1 four, five or six times? Round your answer to 2 decimal places.

Binomial  $n=30$   $p = \frac{1}{6}$   $q = 1-p = \frac{5}{6}$   
 $\uparrow$   
 $p(\text{roll a 1})$

$$P(4, 5, \text{ or } 6 \text{ successes}) = 30C4 \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^{26} + 30C5 \left(\frac{1}{6}\right)^5 \left(\frac{5}{6}\right)^{25} + 30C6 \left(\frac{1}{6}\right)^6 \left(\frac{5}{6}\right)^{24}$$

$$\approx 0.54$$

I also accepted 0.53

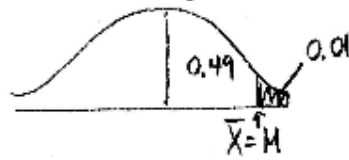
2. [3 marks] Each week there are an average of 5 accidents on a certain stretch of highway. What is the probability of observing at most three accidents on that stretch next week? Round your answer to 2 decimal places.

Poisson  $\mu=5$   $P(X \leq 3) = \left( \frac{5^0}{0!} + \frac{5^1}{1!} + \frac{5^2}{2!} + \frac{5^3}{3!} \right) e^{-5}$   
 $\approx 0.27$

I also accepted 0.26

3. [5 marks] The weight of adult males in Canada is normally distributed with mean 160 lbs and SD  $\sigma = 25$  lbs.

a) Given 10 randomly-chosen adult males in Canada: Find a value  $M$  so that the probability that their weights average to more than  $M$  is 0.01. Round your answer to 2 decimal places.



Reverse look-up  
z

or use formula  
sheet  $z = 2.326$

By Central Limit  
Theorem,

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$\begin{aligned} \mu &= 160 \\ \sigma &= 25 \\ n &= 10 \end{aligned}$$

$$2.33 = \frac{M - 160}{25/\sqrt{10}}$$

$$2.33 \times \frac{25}{\sqrt{10}} = M - 160$$

$$M = 2.33 \times \frac{25}{\sqrt{10}} + 160 \approx 178.42 \text{ lbs.}$$

b) What assumption(s) do you have to check?

The population is normally distributed.

4. [5 marks] A sample of 2200 ball bearings shows that 40 are defective.

a) Find a 95% upper confidence bound for the proportion defective. Round your answer to 3 decimal places.

$$\hat{p} = \frac{40}{2200} \quad n = 2200$$

$$\hat{q} = 1 - \frac{40}{2200} = \frac{2160}{2200}$$

$$p \leq \hat{p} + z_{\alpha} \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$z_{\alpha} = 1.645$$

$$p \leq \frac{40}{2200} + 1.645 \sqrt{\frac{(40 \times 2160)}{2200^3}}$$

$$p \leq 0.023$$

-0.5 if you wrote  $\hat{p}$

I forgot you if you rounded to 2 decimal places.

b) Would a 90% upper confidence bound be larger or smaller than your 95% upper confidence bound from part a)? Explain.

We are less certain that the 90% UCB is at least as big as  $p$ , so the 90% UCB is smaller than 0.023

Alternatively: just calculate the 90% UCB.

5. [3 marks] We want to estimate the population proportion  $p$  with a 95% margin of error of at most 0.02. We don't have any estimates for  $\hat{p}$ . What is the minimum sample size we can use?

$$95\% \text{ M/E} \quad z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}} \leq 0.02$$

$\hat{p}\hat{q}$  can be as big as  
 $0.5 \times 0.5$

$$1.96 \sqrt{\frac{0.5 \times 0.5}{n}} \leq 0.02$$

Use this when  $\hat{p}$   
is unknown.

$$\frac{1.96 \sqrt{0.5 \times 0.5}}{0.02} \leq \sqrt{n}$$

$$\left[ \frac{1.96 \sqrt{0.5 \times 0.5}}{0.02} \right]^2 \leq n$$

$$n \geq 2401$$

6. [6 marks] You are given the following data:  $\bar{x}_1 = 19.26$ ,  $s_1 = 4.25$ ,  $n_1 = 50$ ;  $\bar{x}_2 = 21.30$ ,  $s_2 = 3.75$ ,  $n_2 = 40$ . Test at  $\alpha = 0.01$  whether there is a difference between the two population means.

a) State  $H_0$  and  $H_a$

$$H_0: \mu_1 - \mu_2 = 0 \quad H_a: \mu_1 - \mu_2 \neq 0$$

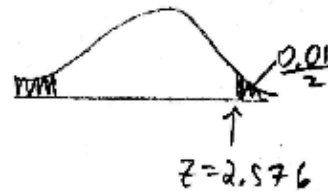
b) What assumption(s) do you need to check?

$$n_1 \geq 30 \quad \checkmark \quad n_2 \geq 30 \quad \checkmark$$

c) Do you reject  $H_0$  or not? Show all your work.

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \quad (D_0 = 0)$$

$$\approx -2.42$$



Don't reject  $H_0$ .

d) Find the p-value of the test.

$$p = P(|z| \geq 2.42)$$

$$= 2 \times (0.5 - 0.4922)$$

$$= 0.0156$$

