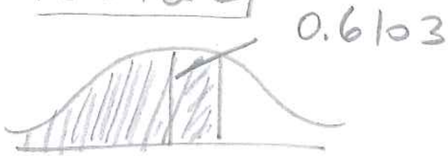
1. [3 marks] The yield (in grams) for a specific chemical reaction is normally distributed with a standard deviation of 16.2 grams. Find the mean yield if 61.03% of these reactions yield less than 91.2 grams.

X = yield (g). X is normal.

$\boxed{\sigma = 16.2}$



$\boxed{X = 91.2}$

$\boxed{z = 0.28}$ 1

$$z = \frac{X - \mu}{\sigma}$$ 1

$$0.28 = \frac{91.2 - \mu}{16.2}$$

$$0.28(16.2) = 91.2 - \mu$$

$$\mu = 91.2 - 0.28(16.2)$$

$$\approx 86.7 \text{ g}$$ 1

2. [4 marks] We want to estimate the population proportion p with a 98% margin of error less than 0.015. We are given $\hat{p} = 0.16$. Find the minimum sample size.

98% M/E < 0,015

$$\begin{cases} \hat{p} = 0.16 \\ \hat{q} = 1 - \hat{p} = 0.84 \end{cases}$$

1

$$z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}} < 0{,}015$$

2

$$\frac{2.326 \sqrt{0.16 \times 0.84}}{\sqrt{n}} < 0{,}015$$

$$\frac{2{,}326 \sqrt{0{,}16 \times 0{,}84}}{0.015} < \sqrt{n}$$

Square both sides:

$$\left[\frac{2.326 \sqrt{0{,}16 \times 0{,}84}}{0.015} \right]^2 < n$$

$$3231.7\ldots < n$$

1

$$\boxed{n = 3232}$$

3. [3 marks] It takes an average of 7.3 seconds to inspect a part, with a standard deviation of 1.3 seconds. Forty parts are randomly selected from the production line. Find the probability that it takes less than 280 seconds in total to inspect all 40 parts.

$\mu = 7.3 \qquad \sigma = 1.3 \qquad n = 40$

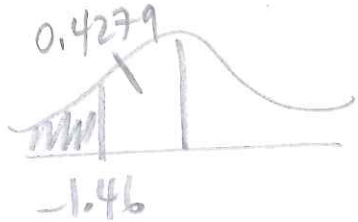
$P(\text{total} < 280)$

$$\left\{ \begin{aligned} \text{total} &= 280 \\ \frac{\text{total}}{40} &= \frac{280}{40} \\ \bar{x} &= 7 \end{aligned} \right.$$

$= P(\bar{x} < 7)$

$$\left\{ \begin{aligned} & n \geq 30 \checkmark \\ & z = \frac{\bar{x} - \mu}{(\sigma / \sqrt{n})} \\ & \quad = \frac{7 - 7.3}{(1.3 / \sqrt{40})} \\ & \quad \approx -1.46 \end{aligned} \right.$$

$= P(z < -1.46)$

$=$ 

$= 0.5 - 0.4279$

$= 0.0721$

4. [4 marks] The random variable X has probability density function

$$f(x) = \begin{cases} \frac{1}{14\sqrt{x}}, & 4 \leq x \leq 81 \\ 0, & \text{otherwise} \end{cases}$$

Find the value of c so that $P(c \leq X \leq 36) = 0.5$

$$P(c \leq X \leq 36) = \int_c^{36} \frac{1}{14\sqrt{x}}\,dx \quad (1)$$

$$= \left[\frac{\sqrt{x}}{7}\right]_c^{36}$$

$$= \frac{6}{7} - \frac{\sqrt{c}}{7} \quad (1)$$

Now set $\frac{6}{7} - \frac{\sqrt{c}}{7} = 0.5$ (1)

$$6 - \sqrt{c} = 3.5$$

$$6 - 3.5 = \sqrt{c}$$

$$2.5 = \sqrt{c}$$

$$c = 6.25 \quad (1)$$

5. [6 marks] Test whether μ_1 is greater than μ_2 at the 2% significance level given the following sample data: $n_1 = 95, \overline{x}_1 = 12.9, s_1 = 2.4, n_2 = 50, \overline{x}_2 = 11.8, s_2 = 3.5$.

-2 if you wrote $\bar{x}_1 = \bar{x}_2$

a) State H_0 and H_a

2 $H_0: \mu_1 = \mu_2$ $(\mu_1 - \mu_2 = 0)$ $H_a: \mu_1 > \mu_2$ $(\mu_1 - \mu_2 > 0)$ right-tailed test

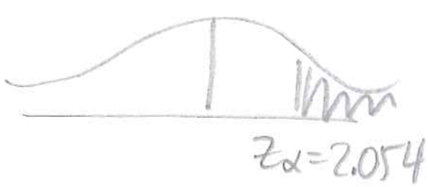
b) Check any necessary assumptions.

1 $n_1, n_2 \geq 30$

c) Do you reject H_0? Show all your work.

$$z = \frac{\bar{x}_1 - \bar{x}_2 - \underbrace{D_0}_{=0 \text{ from } H_0}}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{12.9 - 11.8 - 0}{\sqrt{\frac{2.4^2}{95} + \frac{3.5^2}{50}}}$$

1 ≈ 1.99



1

Don't reject H_0
$\mu_1 = \mu_2$

d) Find the p-value.

$p =$ 

1 $= 0.5 - 0.4767$
$= 0.0233$

6. [5 marks] Find the probability of making a Type II error in the hypothesis test below if the true value of μ is 78.5.

Test $H_0 : \mu = 80.0$ at $\alpha = 0.01$ with $n = 64$, $s = 3.2$.

1) Non-rejection region

$$\mu_0 \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$80.0 \pm 2.576 \left(\frac{3.2}{\sqrt{64}} \right)$$

$$78.97 \le \bar{x} \le 81.03$$

2

2) $P(78.97 \le \bar{x} \le 81.03)$ using $\mu = 78.5$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

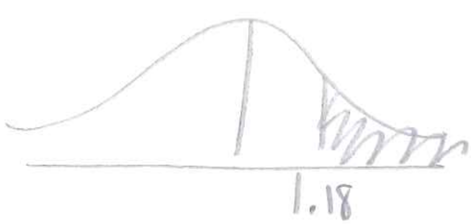
$z_1 \simeq 1.18$

$z_2 \simeq 6.33$

$= P(1.18 \le z \le 6.33)$

2

=



$= 0.5 - 0.3810$

$= 0.1190$

1