PM Section

Math 254 Test Two Total: 25 marks

Name:________________________________

1. [4 marks] The scores on a national exam were normally distributed with a mean of 71 and a population standard deviation of 12 . If a random sample of 40 scores is taken, what is the probability that the mean of the scores is less than 68?

$\mu = 71 \quad \sigma = 12 \quad n = 40$

Since $n \geq 30$, $\bar{x}$ is normally distributed with mean $= 71$ $\quad SE = \frac{\sigma}{\sqrt{n}} = \frac{12}{\sqrt{40}}$

$$P(\bar{x} < 68) = P\left(z < \frac{68-71}{12/\sqrt{40}} \approx -1.58\right)$$

$$= 0.5 - 0.4429$$

$$= 0.0571$$

[In fact, since population is normal, this method works even for small samples since we know σ.]

2. [3 marks] You wish to estimate the population proportion p correct to within 0.005 with probability 90%. You do not have any estimates for p. What is the smallest sample size you should take?

Want 90% $M/E \leq 0.005$

$$z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} \leq 0.005$$

$\hat{p}\hat{q} \leq 0.5 \times 0.5$ from class

$$\Rightarrow \frac{1.645\sqrt{0.5 \times 0.5}}{0.005} \leq \sqrt{n}$$

$$\Rightarrow n \geq 27060.25$$

$\Rightarrow n = 27061.$ is the smallest size.

3. [2 marks] You are presented a 95% confidence interval for μ. Explain what the phrase **95% confidence** means in this context.

95% of sample will produce a CI containing μ.

4. [6 marks] Data from a tire factory's two assembly lines:

Assembly Line	Sample Size	Number of Defective Tires
1	1000	22
2	1200	33

Perfom a hypothesis test at the 5% significance level to test whether Assembly Line 2 has a higher proportion of defective tires.

$H_0: p_1 - p_2 = 0 \qquad H_a: p_2 > p_1$ (1)
one-tailed

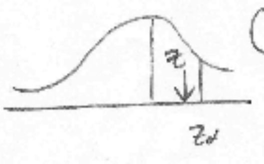
$(n_1\hat{p}_1, n_1\hat{q}_1, n_2\hat{p}_2, n_2\hat{q}_2 > 5 \checkmark)$ (1)

$$Z = \frac{\hat{p}_2 - \hat{p}_1}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} \approx +0.82 \quad (1)$$

$$\hat{p} = \frac{22+33}{1000+1200} = \frac{55}{2200} = 0.025 \quad (1)$$

$$\hat{q} = \frac{2145}{2200}$$

$\alpha = 0.05 \qquad z_\alpha = 1.645$



(1)

Don't reject H_0. (1)

Insufficient evidence that Assembly Line 2 has a higher proportion of defective tires.

5. [6 marks] Weights (in grams) for six Red Delicious apples:

136 180 197 142 165 171

a) You want to test the hypothesis $\mu = 150$ at the 5% significance level. What assumption do you need?

Population of weights is normal. (1)

b) Perform the hypothesis test in part a).

$\boxed{H_0: \mu = 150 \quad H_a: \mu \neq 150 \text{ two-tailed}}$ (1)

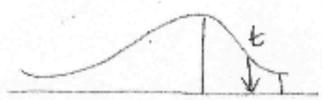
From calc: $\bar{x}$, s

$$t = \frac{\bar{x} - 150}{s/\sqrt{n}}$$

$$\approx 1.612$$ (1)

$\alpha = 0.05$ $\quad t_{\alpha/2} = 2.571$

$df = 5$



(1)

Don't reject H_0. (1)
Insufficient evidence that $\mu \neq 150$.

c) What can you say about the p-value?

$0.05 < P(t > 1.612) < 0.10$

$0.10 < p < 0.20$ from table. (1)

6. [4 marks] Consider H_0: $\mu = 82$ versus H_a: $\mu \neq 82$ tested at the 95% confidence level with a random sample of size 60. The population standard deviation is $\sigma = 8$. If the true population mean is $\mu = 76$, what is the probability of not rejecting H_0?

Find non-rejection region using $\mu = 82$

Find probability using true $\mu = 76$.

Non-rejection: $\bar{x} = 82 \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

$= 82 \pm 1.96 \times \frac{8}{\sqrt{60}}$

$79.98 \leq \bar{x} \leq 84.02$

Find $P(79.98 \leq \bar{x} \leq 84.02 \mid \mu = 76)$

$= P\left(\frac{79.98 - 76}{8/\sqrt{60}} \leq z \leq \frac{84.02 - 76}{8/\sqrt{60}}\right)$

$= P(3.85 \leq z \leq 7.77)$

$\leq 0.50 - 0.4998 = 0.0002$

or $\approx 0\%$ is OK too