

Name: _____

Marks may be deducted for not showing all your work.

1. [4 marks] a) Find the mean and the median of the data set below:

66, 44, 43, 19, 26, 38, 49, 47, 37, 25, 51, 52

$$n = 12$$

$$\text{mean} = \frac{497}{12} \approx 41.4$$

1	9
2	65
3	87
4	4397
5	12
6	6

→

1	9
2	56
3	78
4	<u>34</u> 79
5	12
6	6

$$\text{median} = \frac{43+44}{2} = 43.5$$

- b) Use the mean and the median to decide whether the data is skewed. In which direction, if any, is the data skewed?

$$\text{mean} < \text{median}$$

Mean indicates
direction of skewSkewed to
the left⊖ if you didn't mention
mean, median[More data to the left of the peak value
than to the right].

2. [5 marks] a) A data set has 72 measurements. What does Tchebysheff's Rule say about the **number** of measurements lying in the interval from $\mu - 3\sigma$ to $\mu + 3\sigma$?

$$\# \text{ is } \geq \left(1 - \frac{1}{3^2}\right) 72$$

The number is at least 64.

b) If the data is roughly mound-shaped, what does the Empirical Rule say about the **proportion** of measurements lying in the interval from $\mu - 3\sigma$ to $\mu + 3\sigma$?

$$\approx 99.7\%$$

I'll also accept $\approx 99\%$ or $\approx 100\%$

c) For the data set below, what is the actual proportion of measurements lying in the interval $\mu - 3\sigma \leq X \leq \mu + 3\sigma$?

X	Frequency
3	1
8	12
12	45
16	11
20	3

$$n = 72$$

Calculate μ and σ
on calculator

$$\mu \approx 12.15$$

[Use \bar{x} key]

$$\sigma \approx 2.98$$

$$\mu - 3\sigma \approx 3.21$$

$$\mu + 3\sigma \approx 21.09$$

$$\text{Actual proportion} = \frac{12 + 45 + 11 + 3}{72} \approx 0.99$$

3. [2 marks] A data set consisting of temperatures (in degrees Fahrenheit) has a mean of 68 and a SD of $\sigma = 9$. Each temperature is converted to degrees Celsius by first subtracting 32 and then multiplying by $\frac{5}{9}$. What are the new mean and SD of the data set?

Initially

$$\mu = 68$$

$$\sigma = 9$$

Subtract 32
from each
measurement

$$\mu = 68 - 32 = 36$$

$$\sigma = 9$$

σ does
not change

Multiply each
measurement by
 $\frac{5}{9}$

$$\mu = \frac{5}{9} \times 36 = 20$$

$$\mu = 20$$

$$\sigma = \frac{5}{9} \times 9 = 5$$

$$\sigma = 5$$

4. [3 marks] Find the probability that a 10-bit string begins with 01 or ends with 110.

$$n(S) = 2^{10}$$

$$\begin{aligned} \# \text{ strings beginning with } 01 \text{ or ending with } 110 &= \# \text{ strings } 01 \dots + \# \text{ strings } \dots 110 - \# \text{ strings } 01 \dots 110 \\ &= 2^8 + 2^7 - 2^5 \end{aligned}$$

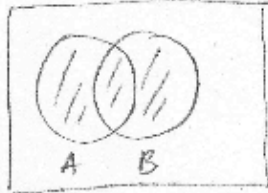
5 bits to choose

$$P(\text{beginning } 01 \text{ or ending } 110) = \frac{2^8 + 2^7 - 2^5}{2^{10}}$$

$$\approx 0.34$$

5. [5 marks] You are given $P(A) = 0.8$, $P(B) = 0.75$ and $P(A \cup B) = 0.91$.

a) Find $P(A \cap B)$, $P(A|B)$ and $P(B|A)$



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.91 = 0.8 + 0.75 - P(A \cap B)$$

$$P(A \cap B) = 0.64$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.64}{0.75} \approx 0.85$$

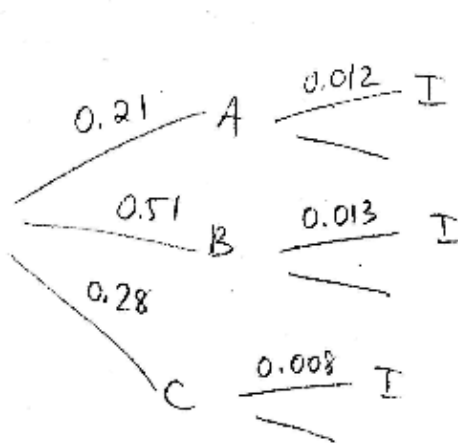
$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{P(A \cap B)}{P(A)} = \frac{0.64}{0.8} = 0.8$$

b) Are A and B independent? Justify your answer.

No. $P(A|B) \neq P(A)$

[Alternatively : $P(B|A) \neq P(B)$
 $P(A \cap B) \neq P(A)P(B)$]

6. [3 marks] Your firm recruits employees from three schools: A, B and C. School A accounts for 21% of recruits; B and C account for 51% and 28% respectively. Of the recruits from school A, 1.2% will be fired for incompetence. The percentages for schools B and C are 1.3% and 0.8% respectively. Given that an employee is fired for incompetence, what is the probability that they were recruited from School C?



I = fired for incompetence

$$P(C|I) = \frac{P(C \cap I)}{P(I)}$$

$$= \frac{0.28 \times 0.008}{0.01139}$$

$$P(I) = 0.21 \times 0.012 + 0.51 \times 0.013 + 0.28 \times 0.008$$

$$= 0.01139$$

$$\approx 0.20$$

7. [3 marks] Your firm employs 25 engineers, 8 administrators and 3 lawyers. How many different committees of six employees consist of either two or three engineers?

25 engineers, 11 others

$$25C2 \times 11C4 + 25C3 \times 11C3$$

2 engineers and 4 others or 3 engineers and 3 others

If you worked out the #: 478,500

Name: _____

Marks may be deducted for not showing all your work.

1. [4 marks] a) Find the mean and the median of the data set below:

44, 66, 67, 91, 84, 72, 61, 63, 73, 85, 59, 58

$n = 12$

$$\text{mean} = \frac{823}{12} \approx 68.6$$

4	4
5	98
6	67 13
7	23
8	45
9	1

→

4	4
5	89
6	13 (67)
7	23
8	45
9	1

$$\begin{aligned} \text{Median} &= \frac{66 + 67}{2} \\ &= 66.5 \end{aligned}$$

- b) Use the mean and the median to decide whether the data is skewed. In which direction, if any, is the data skewed?

$$\text{median} < \text{mean}$$

Mean indicates
direction of skewSkewed to
the right⊕ if you didn't compare
mean and median[More data to the right of the peak value
than to the left.]

2. [5 marks] a) A data set has 72 measurements. What does Chebyshev's Rule say about the **number** of measurements lying in the interval from $\mu - 2\sigma$ to $\mu + 2\sigma$?

$$\# \text{ is } \geq \left(1 - \frac{1}{2^2}\right) 72$$

The number is at least 54.

b) If the data is roughly mound-shaped, what does the Empirical Rule say about the **proportion** of measurements lying in the interval from $\mu - 2\sigma$ to $\mu + 2\sigma$?

Proportion \approx 95%

c) For the data set below, what is the actual proportion of measurements lying in the interval $\mu - 2\sigma \leq X \leq \mu + 2\sigma$?

X	Frequency
3	1
8	12
12	45
16	11
20	3

$$n = 72$$

Find μ and σ on calculator

$$\mu \approx 12.15$$

[Use \bar{x} key]

$$\sigma \approx 2.98$$

$$\mu - 2\sigma \approx 6.19$$

$$\mu + 2\sigma \approx 18.11$$

$$\text{Actual proportion} = \frac{12 + 45 + 11}{72} = \frac{68}{72} \approx 0.94$$

3. [2 marks] A data set consisting of temperatures (in degrees Celcius) has a mean of 12 and a SD of $\sigma = 2$. Each temperature is converted to degrees Fahrenheit by first multiplying by 1.8 and then adding 32. What are the new mean and SD of the data set?

Initially: $\mu = 12$ $\sigma = 2$

Multiply each measurement by 1.8

$$\mu = 12 \times 1.8 = 21.6$$

$$\sigma = 2 \times 1.8 = 3.6$$

Add 32 to each measurement

$$\mu = 21.6 + 32 = 53.6$$

$$\sigma = 3.6$$

No change to σ

$\mu = 53.6$ $\sigma = 3.6$

4. [3 marks] Find the probability that a 12-bit string begins with 01 or ends with 110.

$$n(S) = \# \text{ 12-bit strings} = 2^{12}$$

$$n(A) = \# \text{ strings } 01 \dots + \# \text{ strings } \dots 110 - \# \text{ strings } 01 \dots 110$$

$$= 2^{10} + 2^9 - 2^7$$

$$= 1408$$

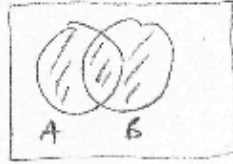
7 bits to be chosen

$$P(\text{begins with } 01 \text{ or ends with } 110) = \frac{1408}{2^{12}}$$

$$\approx 0.34$$

5. [5 marks] You are given $P(A) = 0.3$, $P(B) = 0.23$ and $P(A \cup B) = 0.44$.

a) Find $P(A \cap B)$, $P(A|B)$ and $P(B|A)$



$$\begin{aligned}P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\0.44 &= 0.3 + 0.23 - P(A \cap B) \\P(A \cap B) &= 0.09\end{aligned}$$

$$\begin{aligned}P(A|B) &= \frac{P(A \cap B)}{P(B)} \\&\approx 0.39\end{aligned}$$

$$\begin{aligned}P(B|A) &= \frac{P(B \cap A)}{P(A)} \\&= \frac{P(A \cap B)}{P(A)} \\&= 0.3\end{aligned}$$

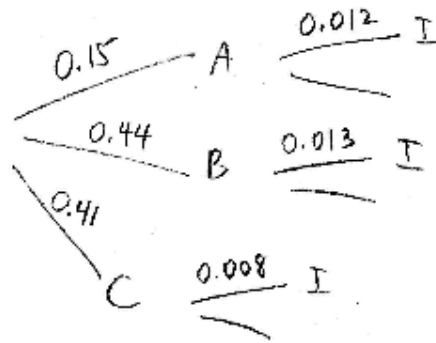
b) Are A and B independent? Justify your answer.

No, because $P(A|B) \neq P(A)$

[Alternatively: $P(B|A) \neq P(B)$
 $P(A \cap B) \neq P(A)P(B)$]

6. [3 marks] Your firm recruits employees from three schools: A, B and C. School A accounts for 15% of recruits; B and C account for 44% and 41% respectively. Of the recruits from school A, 1.2% will be fired for incompetence. The percentages for schools B and C are 1.3% and 0.8% respectively. Given that an employee is fired for incompetence, what is the probability that they were recruited from School C?

I = fired for incompetence



$$P(C|I) = \frac{P(C \cap I)}{P(I)}$$

$$= \frac{0.41 \times 0.008}{0.0108}$$

$$\approx 0.30$$

$$P(I) = 0.15 \times 0.012 + 0.44 \times 0.013 + 0.41 \times 0.008$$

$$= 0.0108$$

7. [3 marks] Your firm employs 18 engineers, 8 administrators and 3 lawyers. How many different committees of six employees consist of either two or three engineers?

18 engineers, 11 others

$${}^{18}C_2 \times {}^{11}C_4 + {}^{18}C_3 \times {}^{11}C_3$$

and 2 engineers and 4 others or 3 engineers and 3 others

If you worked the # out : 185, 130