

1. [3 marks] Consider the following data set.

X	Relative Frequency
125	0.17
145	0.19
165	0.08
185	0.43
205	0.13

a) Calculate the mean.

$$\mu = 125(0.17) + 145(0.19) + \dots + 205(0.13) \\ = 168.2$$

b) Calculate the median.

(the middle measurement)

$$\text{median} = 185$$

c) Based on the mean and the median, in which direction is the data skewed?

$$\text{mean} < \text{median}$$

Data is skewed to the left

2. [4 marks] Consider the following sample: $-6, -1, 2, 7, 8$. Compute the sample standard deviation. Show all your work. Round your answer to two decimal places.

X	$X - \bar{x}$	$(X - \bar{x})^2$
-6	-8	64
-1	-3	9
2	0	0
7	5	25
8	6	36

Sample mean
 $\bar{x} = 2$

Sample Variance

$$s^2 = \frac{64 + 9 + 0 + 25 + 36}{4}$$

$$s^2 = 33.5$$

sample SD $s \approx 5.79$

3. [5 marks] Consider the following data set.

X	Frequency
1	0.05
3	0.42
5	0.49
9	0.04

a) Compute μ and σ using your calculator. Round your values to two decimal places.

$$\mu = 4.12 \quad \sigma \approx 1.53$$

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b) What does Tchebysheff's Theorem predict for the proportion of measurements in $\mu \pm 2.5\sigma$?

$$\geq 1 - \frac{1}{k^2} \text{ of data is } \mu \pm k\sigma$$

$$\geq 1 - \frac{1}{2.5^2} \text{ of data is } \mu \pm 2.5\sigma$$

$$\geq 84\% \text{ of data is } \mu \pm 2.5\sigma$$

2

⊖ for missing/incorrect inequality

c) What is the actual proportion of measurements in $\mu \pm 2.5\sigma$?

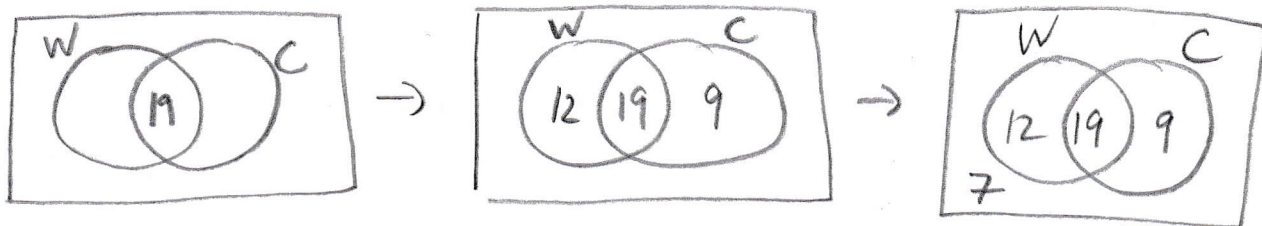
$$\mu \pm 2.5\sigma : 0.295 \leq x \leq 7.945$$

$$\text{actual proportion} = 0.96$$

$$96\%$$

1

4. [2 marks] In a class of 47 students, 31 work and 28 have a car. Of those who have a car, 19 work. What is the probability that a student works or has a car?



$$P(\text{works or car}) = \frac{40}{47} \approx 85\%$$

5. [5 marks] A password consists of 4 digits, where each digit can be chosen from 1, 2, 3, 4, 5. Find the probability that a password:

a) starts with 4

$$n(S) = 5^4 \quad n(A) = 5^3 \quad \boxed{4 \text{ ---}}$$

$$P(A) = \frac{5^3}{5^4} = 0.2$$

b) ends with 55

$$n(A) = 5^2 \quad \boxed{\text{ --- 55}}$$

$$P(A) = \frac{5^2}{5^4} = 0.04$$

c) has no repeated digits

$$n(A) = 5P_4 \text{ or } 5 \times 4 \times 3 \times 2$$

$$P(A) = \frac{5P_4}{5^4} = 0.192$$

d) has at least one repeated digit

$$\begin{aligned} & 1 - P(\text{no repeated}) \\ &= 1 - 0.192 \\ &= 0.808 \end{aligned}$$

6. [3 marks] Are events A and B independent? State yes or no for each scenario.

a) $P(A) = 0.2, P(B) = 0.6, P(A \cap B) = 0.2$

No $P(A \cap B) \neq P(A)P(B)$

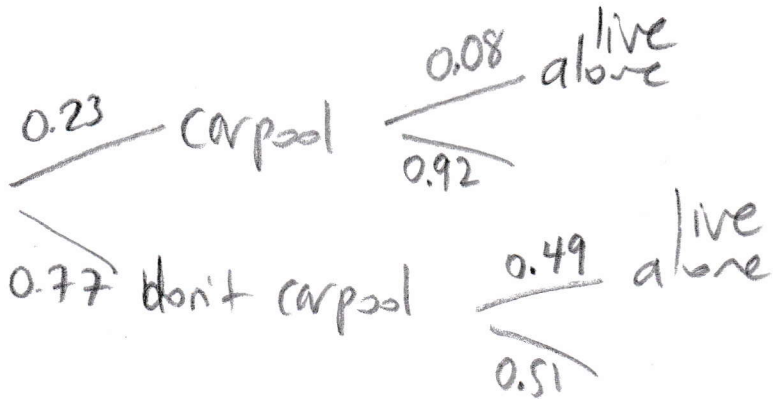
b) $P(A) = 0.2, P(B) = 0.6, P(A \cap B) = 0$

No $P(A \cap B) \neq P(A)P(B)$

c) $P(A) = 0.2, P(B) = 0.6, P(B|A) = 0.6$

YES $P(B|A) = P(B)$

7. [3 marks] 23% of students carpool. Of the students who carpool, 8% live alone. Of the students who don't carpool, 49% live alone. Find the probability that a student who lives alone carools. Round your answer to two decimal places.



$$\begin{aligned} P(\text{carpool} | \text{lives alone}) &= \frac{P(\text{carpool and alone})}{P(\text{alone})} \\ &= \frac{0.23(0.08)}{[0.23(0.08) + 0.77(0.49)]} \\ &\approx 0.05 \end{aligned}$$

8. [5 marks] A shipment contains 33 good and 7 defective parts. Three parts are randomly selected from the shipment. Let X be the number of defective parts selected.

a) Find the probability distribution of X . Round the probabilities to three decimal places.

33G
7D

$X = \#D$	Outcome	# ways	$P(X)$
0	3G	${}^{33}C_3 = 5456$	0.552
1	2G and 1D	${}^{33}C_2 \times {}^7C_1 = 3696$	0.374
2	1G and 2D	${}^{33}C_1 \times {}^7C_2 = 693$	0.070
3	3D	${}^7C_3 = 35$	0.004
		<u><u>9880</u></u>	

b) Find the expected value of X

$$\begin{aligned} \mu \text{ or } E(X) &= 0(0.552) + 1(0.374) + 2(0.070) \\ &\quad + 3(0.004) \\ &= 0.526 \end{aligned}$$