AFTERNOON SECTION

Math 254 Test One

Time: 50 Minutes Total: 30 marks

Name:_____

1. [6 marks] For the following data set: 3, 3, 3, 4, 4, 4, 4, 4, 12

a) State the mean μ and the standard deviation σ (use your calculator)

$\mu \approx 4.56 \quad \sigma \approx 2.67$

b) What proportion of measurements lie within the interval $\mu \pm 3\sigma$?

The interval is from -3.45 to 12.57

$\Rightarrow$ 100% of measurements

c) What does Tchebyshaff's Theorem state about that the proportion of measurements lying in the interval $\mu + 3\sigma$?

The proportion is at least $1 - 1/3^2 = 8/9$

d) If the distribution of a data set is mound-shaped, what does the Empirical Rule state about the proportion of measurements in the interval $\mu \pm 3\sigma$?

The proportion is (approximately) 0.997

2. [2 marks] The average time between subway trains during rush hour in a certain North American city is 2.2 minutes. If you are told that the set of times is skewed, would you expect it to be skewed to the right or left? Explain why. Skewed to the right. Because times must be ≥ 0 we expect large outliers but not small ones.

Note: Set of times is the set of waiting times during rush hour. However, if you misinterpret the set as being "all wait times", the set is still right skewed.

3. [4 marks] A major airport has an average of four near-misses on takeoffs and landings in each one year period. What is the probability that they will have at least two near-misses in the next one year period?

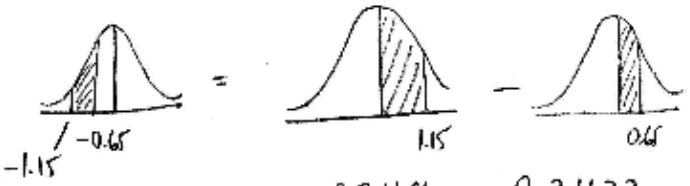
Poisson let X = # near-misses each year with $\lambda = 4$.

$$P(X \geq 2) = 1 - P(X \leq 1)$$
$$= 1 - P(X=1) - P(X=0)$$
$$= 1 - \frac{e^{-4}4^1}{1!} - \frac{e^{-4}4^0}{0!}$$
$$\approx 0.908$$

4. [4 marks] At a chocolate bar factory, the weight of a certain brand of chocolate bar is normally distributed with a mean of 85.03 grams and a standard deviation of 0.2 grams. Calculate the probability that a chocolate bar weighs between 84.80 and 84.90 grams.

$\mu = 85.03 \quad \sigma = 0.2$

$z_1 = \frac{84.80 - \mu}{\sigma} = -1.15 \qquad z_2 = \frac{84.90 - \mu}{\sigma} = -0.65$



$= 0.3749 - 0.2422$

$= 0.1327$

5. [4 marks] A team of 4 people is formed from 16 engineers, including Doug, Ellen, Fred and Greg.

a) How many ways are there to form the team if at most one of Doug, Ellen, Fred, and Greg are selected?

$4C1 \times 12C3 + 4C0 \times 12C4 = 1375$

Select one of D,E,F,G and 3 others OR Select None of D,E,F,G and 4 others

b) If the team members are assigned the roles of Project Lead, Project Liaison, Project Secretary and Project Treasurer (all held by different people), what is the probability that these roles are assigned to Doug, Ellen, Fred and Greg in that order?

Sample space = {all possible assignments}

$\boxed{16}_{\text{Lead}} \times \boxed{15}_{\text{Liaison}} \times \boxed{14}_{\text{Secretary}} \times \boxed{13}_{\text{Treasurer}}$

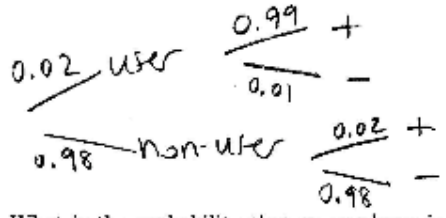
$= 16P4$

ways to assign = 1

$P(\text{given assignment}) = \frac{1}{16P4}$

6. [4 marks] At your engineering firm, 2% of your employees are using steroids. In the yearly drug test users test positive 99% of the time and non-users test negative 98% of the time.

a) Draw a tree diagram representing the situation.



b) What is the probability that an employee is a non-user given that they test positive?

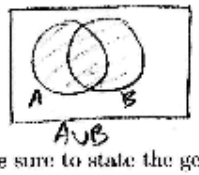
Bayes' Rule

$$P(\text{non-user} \mid +) = \frac{P(\text{non-user} \wedge +)}{P(+)}$$

$$= \frac{0.98 \times 0.02}{[0.98 \times 0.02 + 0.02 \times 0.99]} \approx 0.497$$

7. [6 marks] For two events A and B: $P(A) = 0.35, P(B) = 0.40$ and $P(A \cup B) = 0.60$

a) Calculate $P(A \cap B)$



$P(A \cup B) = P(A) + P(B) - P(A \wedge B)$

$0.6 = 0.35 + 0.4 - P(A \wedge B)$

$\Rightarrow P(A \wedge B) = 0.15$

b) Calculate: $P(A|B)$ and $P(B|A)$ (Be sure to state the general formulas.)

$$P(A|B) = \frac{P(A \wedge B)}{P(B)} = 0.375 \qquad P(B|A) = \frac{P(A \wedge B)}{P(A)} \approx 0.429$$

c) Are A and B independent? (Be sure to state which probabilities you are comparing).

No, because $P(A|B) \neq P(A)$

[or $P(B|A) \neq P(B)$

or $P(A \wedge B) \neq P(A) \cdot P(B)$]