1. Poisson $\mu = 2 / \text{second}$ $X = \#\text{requests}/\text{second}$

$= 6 / 3 \text{ seconds}$

$$P(X \geq 5) = 1 - P(X \leq 4)$$

$$= 1 - [P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)]$$

$$= 1 - \left[e^{-6}\frac{6^0}{0!} + \cdots + e^{-6}\frac{6^4}{4!} \right]$$

$$= 1 - e^{-6}\left[\frac{6^0}{0!} + \frac{6^1}{1!} + \frac{6^2}{2!} + \frac{6^3}{3!} + \frac{6^4}{4!} \right]$$

$$\approx 0.715$$

2.

X	Outcome	# Outcomes	P(X)
0	3 non-spades	39C3	0.414
1	1 spade and 2 non-spades	13C1 × 39C2	0.436
2	2 spades and 1 non-spade	13C2 × 39C1	0.138
3	3 spades	13C3	0.013
		22100 or 52C3	

3

X = # answers correct

Binomial $\quad n = 150 \quad p = \frac{1}{5} = 0.2 \quad q = 1 - p = 0.8$

$P(15 \le X \le 25)$

$np, nq > 5$ ✓

$\mu = np = 30$

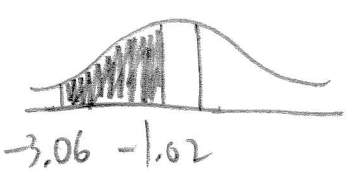
$\sigma = \sqrt{npq} \approx 4.8990$

$z_1 = \frac{25 - \mu}{\sigma} \approx -1.02$

$z_2 = \frac{15 - \mu}{\sigma} \approx -3.06$

$= P(-3.06 \le z \le -1.02)$

$=$



$= 0.4989 - 0.3461$

$= 0.1528$

(4) $P(X>4.5 \mid X>3)$

$$= \frac{P(X>4.5 \text{ and } X>3)}{P(X>3)}$$

$$= \frac{P(X>4.5)}{P(X>3)}$$

$$= \frac{\int_{4.5}^{\infty} 4e^{-4x}\,dx}{\int_{3}^{\infty} 4e^{-4x}\,dx}$$

$$= \frac{\lim_{a\to\infty} \left[-e^{-4x}\right]_{4.5}^{a}}{\lim_{b\to\infty} \left[-e^{-4x}\right]_{3}^{b}}$$

$$= \frac{\lim_{a\to\infty} -e^{-4a} + e^{-18}}{\lim_{b\to\infty} -e^{-4b} + e^{-12}}$$

$$= \frac{e^{-18}}{e^{-12}}$$

$$= e^{-6}$$

$$\approx 0.002$$

(5)

99% M/E ≤ 0.05

$$z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} \le 0.05$$

$\alpha = 0.01$
$z_{\alpha/2} = 2.576$

No info about $\hat{p}$:
use $\hat{p} = 0.5 = \hat{q}$

$$\frac{2.576\sqrt{0.5 \times 0.5}}{\sqrt{n}} \le 0.05$$

$$\frac{2.576\sqrt{0.5 \times 0.5}}{0.05} \le \sqrt{n}$$

$$\left(\frac{2.576\sqrt{0.5 \times 0.5}}{0.05}\right)^2 \le n$$

$$n \ge 663.5776$$

$$\boxed{n \ge 664}$$

(6) $p = 0.02 \qquad q = 1 - p = 0.98$

$n = 400 \qquad \hat{p} = \text{sample proportion}$

$P(6 \leq \#\text{defective} \leq 9)$

$= P\left(\frac{6}{400} \leq \hat{p} \leq \frac{9}{400}\right)$

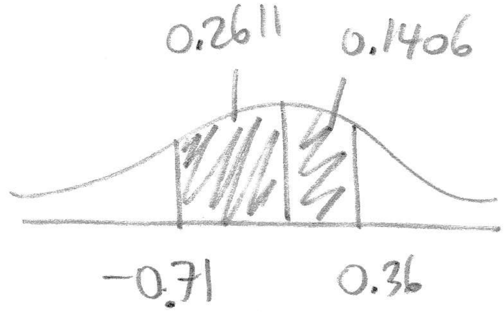
$np, nq > 5$ ✓

$$z_1 = \frac{\frac{6}{400} - p}{\sqrt{\left(\frac{pq}{n}\right)}} \simeq -0.71$$

$$z_2 = \frac{\frac{9}{400} - p}{\sqrt{\left(\frac{pq}{n}\right)}} \simeq 0.36$$

$= P(-0.71 \leq z \leq 0.36)$

$=$



$= 0.4017$

⑦ 1) $H_0: p_1 = p_2$ $\qquad$ $H_a: p_1 < p_2$

or $p_1 - p_2 < 0$

left-tailed

2) Assumptions:

$n_1\hat{p}_1,\ n_1\hat{q}_1,\ n_2\hat{p}_2,\ n_2\hat{q}_2$ all > 5 $\qquad$ $\hat{q}_1 = 0.94$

$\hat{q}_2 = 0.92$

3) $$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

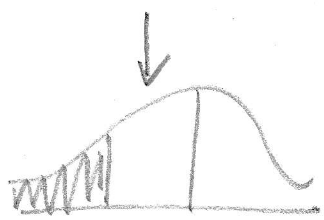
$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2} = 0.072$$

$$\hat{q} = 1 - \hat{p} = 0.928$$

$$= \frac{0.06 - 0.08}{\sqrt{\left(0.072(0.928)\left(\frac{1}{200} + \frac{1}{300}\right)\right)}}$$

≈ -0.85

4) Rejection Region



$-z_\alpha$
$= -1.645$

$\alpha = 0.05$

5) $\boxed{\text{Don't Reject } H_0}$

6) p-value: $P(z < -0.85)$

$=$ 

-0.85

$= 0.5 - 0.3023$

$= 0.1977$

(8) Non-rejection region:

$$\mu_0 \pm z_{\alpha/2} \frac{s}{\sqrt{n}}$$

$H_0: \mu = 200$ (μ_0)

$n = 64 \quad s = 16$

$\alpha = 0.05 \quad z_{\alpha/2} = 1.96$

$$200 \pm 1.96 \left(\frac{16}{\sqrt{64}}\right)$$

$$196.08 \le \bar{x} \le 203.92$$

Now find $P(196.08 \le \bar{x} \le 203.92 \mid \mu = 205)$

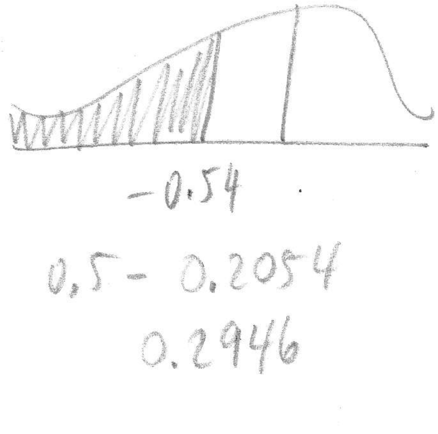
$$z = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$z_1 = \frac{196.08 - 205}{16/\sqrt{64}} = -4.46$$

$$z_2 = \frac{203.92 - 205}{16/\sqrt{64}} = -0.54$$

$= P(-4.46 \le z \le -0.54)$

$=$



$= 0.5 - 0.2054$

$= 0.2946$